11. Single-phase AC Controller with R-L load

The single –phase AC controller with resistive load is shown in Fig.11.1.Due to the inductance in the circuit , the current in thyristor T1 would not fall to zero at ωt = π, when the source voltage v_s start to be negative. Thyristor T1 continues to conduct until its current i1 falls to zero at ωt =ß.

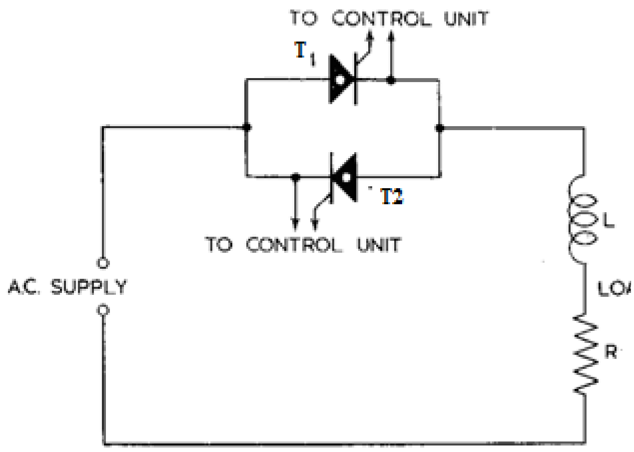
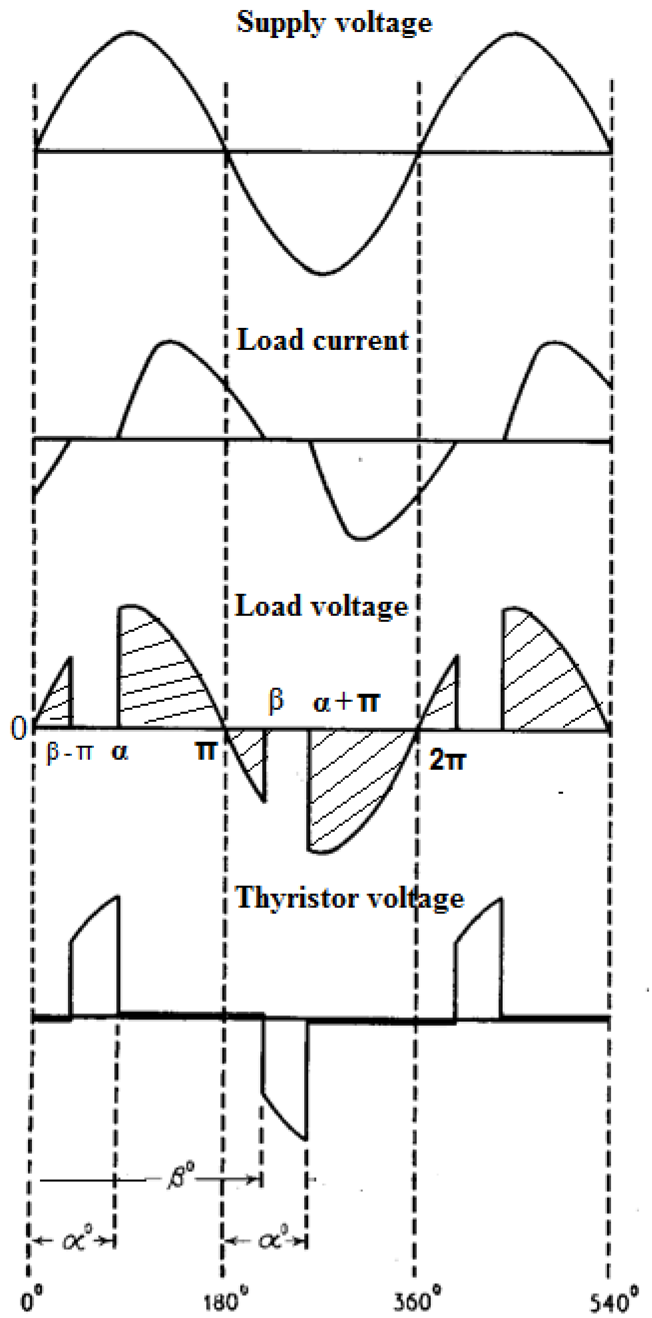


Fig.1



By KVL;

$v_s = v_o = V_R + v_L$

The equation for the current through R-L load can be found from the solution of the differential equation:

$$L\frac{di}{dt} + i\ R = V_m \sin \omega t$$

The solution of this differential equation is:

- The start of conduction is delayed until $\omega t = \alpha$. Subsequent to triggering, let the instantaneous current $i(\omega t)$ consists of hypothetical steady-state components $i_{ss}(\omega t)$ and transient component $i_{trans}(\omega t)$,

$$\therefore i(\omega t) = i_{ss}(\omega t) + i_{trans}(\omega t)$$

Now at $\omega t = \alpha$, the instantaneous steady-state component has the value,

$$i_{ss}(\alpha) = \frac{V_m}{Z}\sin(\alpha - \theta)$$

where

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

The transient component of the current,

$$i_{trans} = Ae^{-\frac{R}{L}t}$$

Hence the equation for the current i,

$$i(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\frac{R}{L}t} \quad \alpha \le \omega t \le \beta$$

The constant A can be found from the initial conditions:

At $\omega t = \alpha$, $i = 0$, hence from eq.(2) ,

$$A = -\frac{V_m}{Z}\sin(\alpha - \theta)e^{-(\frac{R}{L})(\frac{\alpha}{\omega})}$$

- <u>Subsequent to</u> α

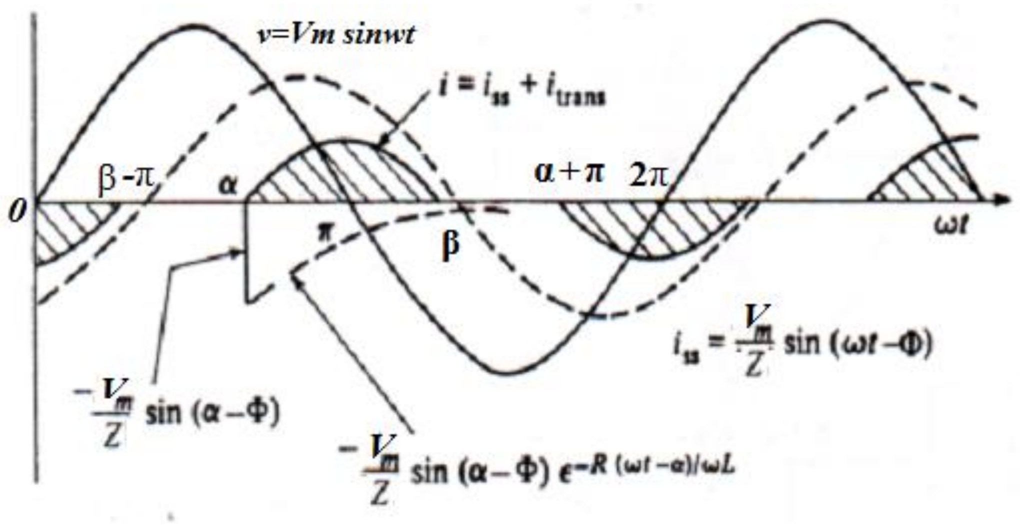
The transient component decay exponentially from it's instantaneous value of [$-\frac{V_m}{Z}\sin(\alpha - \theta)$] by the time constant $\tau = \frac{L}{R.}$. Thus

$$i_{trans}(\omega t) = \frac{-V_m}{Z}\sin(\alpha - \theta)e^{-\frac{t}{\tau}}$$

For $\omega t > \alpha$, $$i_{trans}(\omega t) = -\frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\omega t - \alpha)}$$

- <u>The complete solution for the current at first cycle</u>

$$i(\omega t) = \frac{v_m}{Z}\sin(\omega t - \theta) - \frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\omega t - \alpha)}$$



Extinction Angle β

For the current in the interval $\alpha \le \omega t \le \beta$

$$i(\omega t) = \frac{v_m}{Z}\sin(\omega t - \theta) - \frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\omega t - \alpha)}$$

But at $\omega t = \beta, i(\omega t) = 0$, hence

$$0 = \frac{v_m}{Z}\sin(\beta - \theta) - \frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\beta - \alpha)}$$

But $\frac{R}{\omega L} = \cot\theta$

$$\therefore 0 = \sin(\beta - \theta) - \sin(\alpha - \phi)e^{-\cot\theta(\beta - \alpha)}$$

If α and θ are known, β can be calculated. However, this is a transcendental equation (i.e. cannot be solved explicitly and no way of obtaining $\beta = f(\alpha, \theta)$).

Method of solution is by iteration,

e.g. If $\theta = 60^0$, $\alpha = 120^0 = 2\pi / 3$

$$\cot\theta = 0.578$$

$$\sin(\beta - 60^0) = \sin(120^0 - 60)e^{-0.578(\beta - \frac{2\pi}{3})} \Rightarrow \beta = 222^0$$

<u>Approximate solution of β :-</u>

$$\beta = 180^0 + \theta - \Delta$$

where

$$\begin{aligned} \Delta &= 5^0 \sim 10^0 \text{ for large R and small } \omega L \\ &= 10^0 \sim 15^0 \text{ for } R = \omega L \\ &= 15^0 \sim 20^0 \text{ for small R and large } \omega L \end{aligned}$$

For the previous example,

$$\theta = 60^0, \Delta = 15^0 - 20^0$$

$$\therefore \beta = 180^0 + 60^0 - 15^0 = 225^0$$

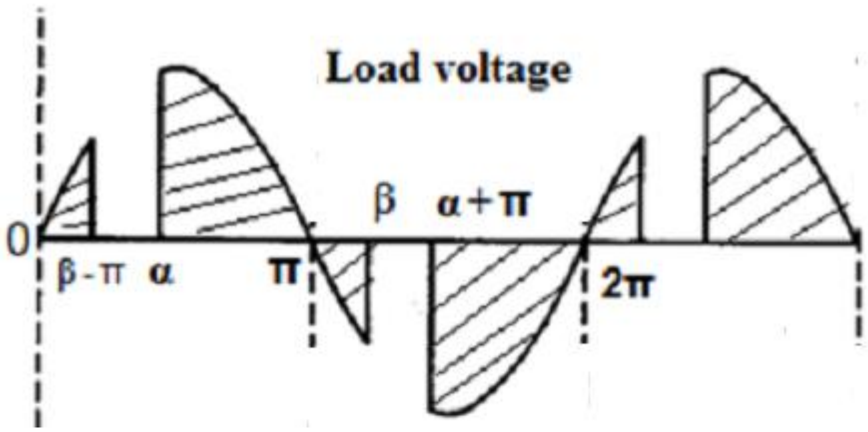
$$or$$

$$\beta = 180^0 + 60^0 - 20^0 = 220^0$$

$$\beta = \frac{225+220}{2} = 222.5^{o}$$

Load voltage waveform analysis

The load voltage waveform for R-L load with AC chopper is shown in Fig.



This waveform can be represented as :

$$v_L(\omega t) = V_m \sin \omega t \Big|_{0,\alpha,\pi+\alpha}^{\beta-\pi,\ \beta,\ 2\pi}$$

$= 0$ elsewhere

Fourier analysis of the above waveform,

$$a_o = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t) d\omega t = 0$$

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\ cos\omega t\ d\omega t$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} V_m sin\omega t \; cos\omega t \, d\omega t$$

$$a_1 = \frac{1}{\pi}\int_0^{\beta-\pi} V_m sin\omega t \; cos\omega t \, d\omega t + \frac{1}{\pi}\int_\alpha^{\beta} V_m sin\omega t \; cos\omega t \, d\omega t + \frac{1}{\pi}\int_{\alpha+\pi}^{2\pi} V_m sin\omega t \; cos\omega t \, d\omega t$$

Use the trigonometric relation sin2x =2 sinx cosx

$$a_1 = \frac{V_m}{2\pi}[\int_0^{\beta-\pi} sin2\omega t \; d\omega t + \int_\alpha^{\beta} sin2\omega t \; d\omega t + \int_{\alpha+\pi}^{2\pi} sin\omega t \; d\omega t]$$

$$a_1 = \frac{V_m}{4\pi}\left[-cos2\omega t \left\{ \begin{matrix} \beta-\pi \\ 0 \end{matrix} \right. - cos2\omega t \left\{ \begin{matrix} \beta \\ \alpha \end{matrix} \right. - cos2\omega t \left\{ \begin{matrix} 2\pi \\ \pi+\alpha \end{matrix} \right. \right]$$

$$a_1 = \frac{V_m}{2\pi}[\cos 2\alpha - \cos 2\beta]$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t) \; sin\omega t \, d\omega t$$

$$b_1 = \frac{1}{\pi}\int_0^{\beta-\pi} V_m sin\omega t \; sin\omega t \, d\omega t + \frac{1}{\pi}\int_\alpha^{\beta} V_m sin\omega t \; sin\omega t \, d\omega t + \frac{1}{\pi}\int_{\alpha+\pi}^{2\pi} V_m sin\omega t \; sin\,\omega t \, d\omega t$$

$$b_1 = \frac{V_m}{2\pi}[2(\beta-\alpha) - \sin 2\beta + \sin 2\alpha]$$

Worked Examples

Example 1: An ideal single-phase voltage source $V_s = V_m \sin\omega t$ supplies to a series R-L load of phase-angle $\theta = 60^\circ$ via a pair of inverse-parallel connected SCRs. The SCR firing angle $\alpha = 120^\circ$. Calculate the current extinction angle β by an approximate method and use iteration to obtain the accurate value.

Solution

(1) By approximate method $\beta \cong \pi + \theta - \Delta$, For $\theta = 60^\circ$, $\Delta \simeq 15^\circ$.

$\therefore \beta = 180 + 60 - 15 = 220^\circ$.

(2) By the accurate method :

$$\sin(\beta - \theta) = \sin(\alpha - \theta)\, e^{(R/L)\left(\frac{\alpha - \beta}{\omega}\right)}$$

or

$$\sin(\beta - \theta) = \sin(\alpha - \theta)\, e^{-\cot\theta\,(\beta - \alpha)}$$

where: $\frac{R}{\omega L} = \cot\theta = \cot 60^\circ = \frac{1}{\tan\theta} = 0.58$.

$\alpha = 120^\circ = \frac{2\pi}{3} = 2.09$ rad.

$\sin(\alpha - \theta) = \sin(120 - 60) = \sin 60 = 0.866$.

Iterative Evaluation of β.

β degrees	β rad	$\sin(\beta - 60^\circ)$ (LHS)	$\sin(\alpha - \theta)\, e^{-\cot\theta(\beta-\alpha)}$
220	3.84	0.342	0.314
222	3.87	0.309	0.308

Example 2 :

Two SCRs are connected in inverse-parallel for the voltage control of a single-phase series R-L load of phase angle $\theta = 45^\circ$. These are each triggered at a firing-angle $\alpha = 90^\circ$ with reference to thier respective anode voltage waveforms. Estimate (do not calculate) the current extinction angle and use this to calculate the load voltage displacement factor.

Solution :

The current extinction angle β is :

$$\beta = \pi + \theta - \Delta$$

with $\theta = 45^\circ$, $\Delta \cong 10^\circ$

$$\therefore \ \beta = \pi + 45 - 10 = 180 + 45 - 10 = 215^\circ \ \text{ or } 3.75 \text{ rad.}$$

$$a_1 = \frac{V_m}{2\pi}(\cos 2\alpha - \cos 2\beta)$$

$$= \frac{V_m}{2\pi}(-1 - 0.342) = \frac{-V_m}{2\pi}(1.342)$$

$$b_1 = \frac{V_m}{2\pi}\left[2(\beta - \alpha) - \sin 2\beta + \sin 2\alpha\right]$$

$$= \frac{V_m}{2\pi}\left[2(3.75 - 1.57) - 0.94 + 0\right]$$

$$= \frac{V_m}{2\pi}(3.42)$$

$$\psi_1 = \tan^{-1}\frac{a_1}{b_1} = \tan^{-1}(-0.392) = -21.4^\circ.$$

$$\text{Displacement Factor} = \cos\psi_1 = \cos 21.4^\circ = 0.93.$$