DTFS

Example: Determine the Fourier series representation of the following discrete time signal and sketch the frequency spectrum.

$$x(n) = \{\ldots, 1, 2, -3, 1, 2, -3, 1, 2, -3, \ldots\}$$

Solution:

A one period three samples 1, 2, -3

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$= \frac{1}{3} \sum_{n=0}^{2} x(n)\, e^{-j\frac{2\pi kn}{3}}$$

$x(0) = 1$, $x(1) = 2$, $x(2) = -3$

$$k=0, \quad C_{k=0} = \frac{1}{3}\left[x(0)\, e^{-j\frac{2\pi (0)(0)}{3}} + x(1)\, e^{-j\frac{2\pi (0)(1)}{3}} + x(2)\, e^{-j\frac{2\pi (0)(2)}{3}}\right]$$

$$= \frac{1}{3}[1 + 2 + (-3)] = 0$$

$$\boxed{C_0 = 0}$$

$$C_{k=1} = \frac{1}{3}\left[1\, e^{-j\frac{2\pi (1)(0)}{3}} + 2\, e^{-j\frac{2\pi (1)(1)}{3}} + \cdots\right]$$

$$C_1 = 0.5 - j\frac{5\sqrt{3}}{6}, \quad C_1 = 1.527 \angle -1.24 \text{ rad}$$

$$C_2 = 0.5 + j\,1.443 = 1.527 \angle 1.24 \text{ rad}$$

DTFS

$$x(n) = \sum_{k=0}^{N-1} C_k e^{\frac{j2\pi kn}{N}} = \sum_{k=0}^{2} C_k e^{\frac{j2\pi kn}{3}}$$

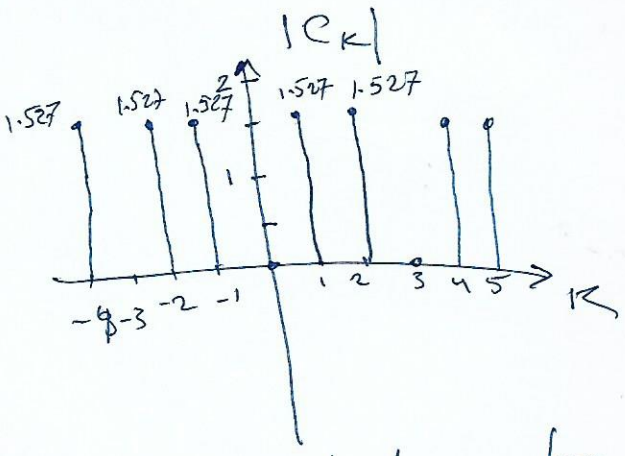
$$x(n) = C_0 e^{\frac{j2\pi(0)n}{3}} + C_1 e^{\frac{j2\pi(1)n}{3}} + C_2 e^{\frac{j2\pi(2)n}{3}}$$

$$x(n) = \left(1.527 e^{-j0.395\pi} + 1.527 e^{j0.395\pi}\right) e^{j2\omega_0 n}$$

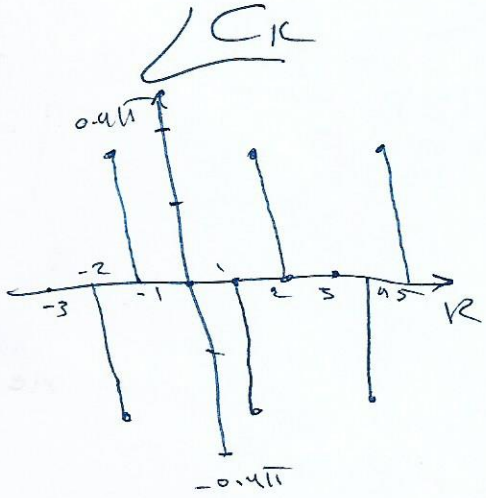
$$C_k = \{C_0, C_1, C_2\} = \{0,\ 1.527\angle -0.395\pi,\ 1.527\angle 0.395\pi\}$$

Magnitude spectrum, $|C_k| = \{0, 1.527, 1.527\}$

Phase spectrum, $\angle C_k = \{0, -0.395\pi, 0.395\pi\}$



Magnitud spectrum



phase spectrum