

Poles and Zeros of Rational Function of Z

(26)
ZT

$$X(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

$$X(z) = \frac{b_0 \left(\frac{b_0}{b_0} + \frac{b_1}{b_0} z^{-1} + \frac{b_2}{b_0} z^{-2} + \dots + \frac{b_M}{b_0} z^{-M} \right)}{a_0 \left(\frac{a_0}{a_0} + \frac{a_1}{a_0} z^{-1} + \frac{a_2}{a_0} z^{-2} + \dots + \frac{a_N}{a_0} z^{-N} \right)}$$

$$G_1 = \frac{b_0}{a_0}$$

$$X(z) = G_1 \frac{z^{-M} \left(z^M + \frac{b_1}{b_0} z^{M-1} + \frac{b_2}{b_0} z^{M-2} + \dots + \frac{b_M}{b_0} \right)}{z^{-N} \left(z^N + \frac{a_1}{a_0} z^{N-1} + \frac{a_2}{a_0} z^{N-2} + \dots + \frac{a_N}{a_0} \right)}$$

$$X(z) = \frac{0.5 - 0.4z^{-1} + 0.06z^{-2}}{2 + 1.6z^{-1} + 0.64z^{-2}}$$

$$X(z) = \frac{0.5 \left(\frac{0.5}{0.5} - \frac{0.4}{0.5} z^{-1} + \frac{0.06}{0.5} z^{-2} \right)}{2 \left(\frac{2}{2} + \frac{1.6}{2} z^{-1} + \frac{0.64}{2} z^{-2} \right)}$$

$$G_1 = \frac{0.5}{2} = 0.25$$

$$X(z) = \frac{0.25 z^{-2} (z^2 - 0.8z + 0.12)}{z^{-2} (z^2 + 0.8z + 0.32)}$$

$$X(z) = \frac{0.25 (z^2 - 0.8z + 0.12)}{(z^2 + 0.8z + 0.32)}$$

$$X(z) = G \frac{z^{-M} \left(z^M + \frac{b_1}{b_0} z^{M-1} + \frac{b_2}{b_0} z^{M-2} + \dots + \frac{b_M}{b_0} \right)}{z^{-N} \left(z^N + \frac{a_1}{a_0} z^{N-1} + \frac{a_2}{a_0} z^{N-2} + \dots + \frac{a_N}{a_0} \right)}$$

27
ZT

$$X(z) = \frac{0.25 (z^2 - 0.8z + 0.12)}{(z^2 + 0.8z + 0.32)}$$

roots
zeros

$$X(z) = G \frac{(z - z_1)(z - z_2) \dots (z - z_M)}{(z - p_1)(z - p_2) \dots (z - p_N)}$$

poles roots

Zeros $\rightarrow X(z) \Rightarrow 0$
 Poles $\rightarrow X(z) \Rightarrow \infty$

Quadratic Equation

$$az^2 + bz + c = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a(z - z_1)(z - z_2) = 0$$

$$a(z - p_1)$$

$$X(z) = \frac{0.25 (z^2 - 0.8z + 0.12)}{(z^2 + 0.8z + 0.32)}$$

(28)
ZT

$$(z^2 + 0.8z + 0.32)$$

$$z^2 - 0.8z + 0.12$$

$$a = 1$$

$$b = -0.8$$

$$c = 0.12$$

$$z = \frac{0.8 \pm \sqrt{0.8^2 - 4 \times 0.12}}{2}$$

$$z_1 = 0.6$$

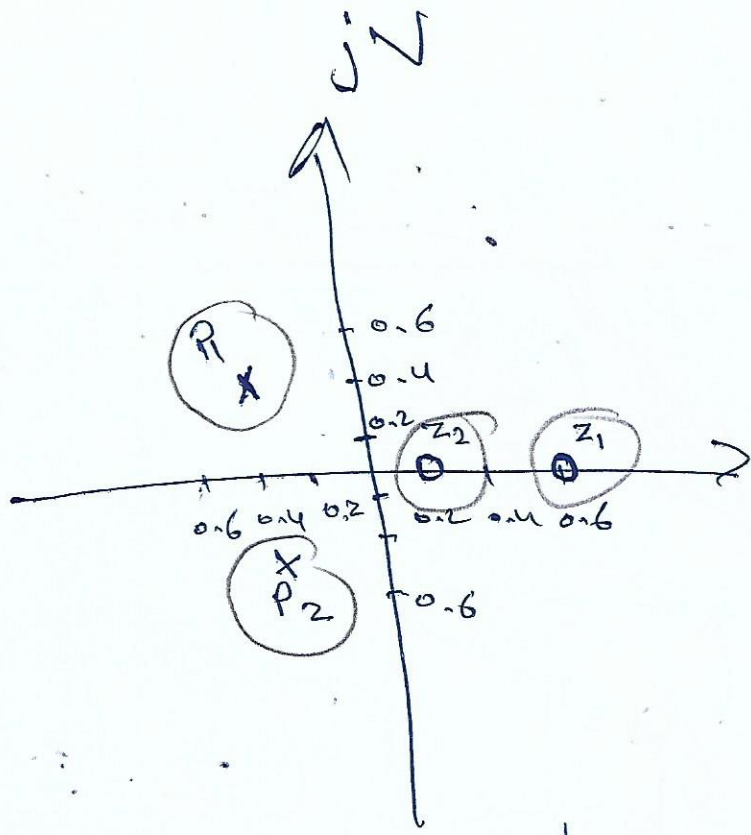
$$z_2 = 0.2$$

$$z^2 - 0.8z + 0.12 = (z - 0.6)(z - 0.2)$$

$$z^2 + 0.8z + 0.32 = (z + 0.4 - j0.4) (z + 0.4 + j0.4)$$

$$X(z) = \frac{0.25 (z - 0.6)(z - 0.2)}{(z + 0.4 - j0.4)(z + 0.4 + j0.4)}$$

Zeros $\left\{ \begin{array}{l} z_1 = 0.6 \\ z_2 = 0.2 \end{array} \right\}$ Poles $\left\{ \begin{array}{l} P_1 = -0.4 + j0.4 \\ P_2 = -0.4 - j0.4 \end{array} \right\}$



(29)
ZT

z-plane

4