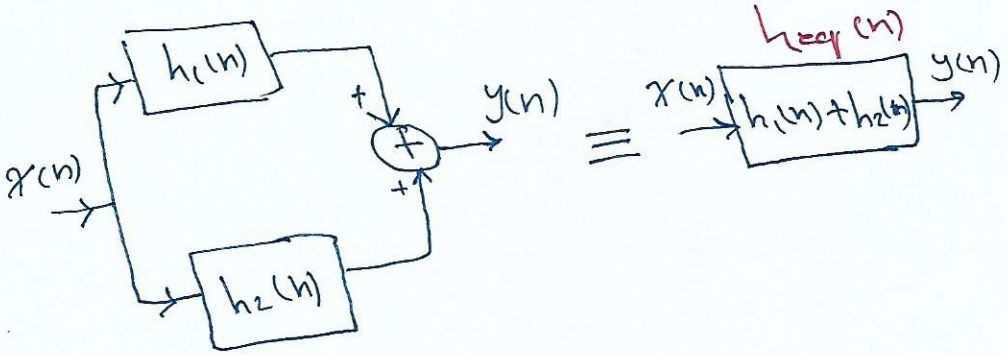
3. Distributive Property

The distributive property of the convolution operator states that

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

From a system point of view, the property of distributive states that if two systems with impulse responses $h_1(n)$ and $h_2(n)$ are connected in parallel, an equivalent system is one that has an impulse response equal to the sum of $h_1(n)$ and $h_2(n)$:

$$h_{eq}(n) = h_1(n) + h_2(n)$$



H.W:

Given the following two sequences

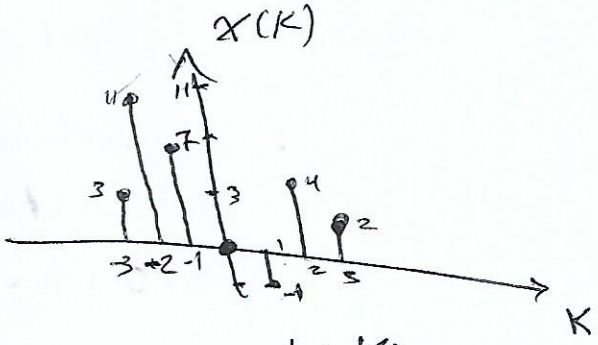
$$x(n) = [3, 11, 7, \underset{\uparrow}{0}, -1, 4, 2] \quad -3 \le n \le 3$$

$$h(n) = [2, \underset{\uparrow}{3}, 0, -5, 2, 1] \quad -1 \le n \le 4$$

determine the convolution $y(n) = x(n) * h(n)$

solution

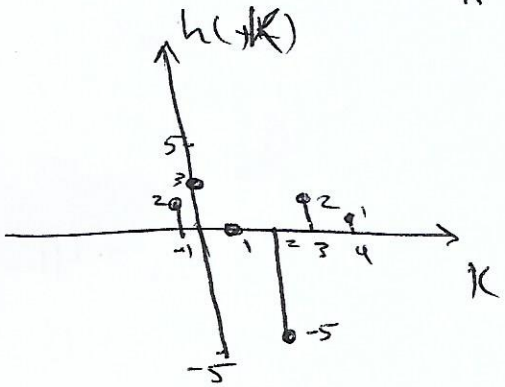
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$



The begining point of n is

min time index of input signal + min time index of impulse response

$$-3 + (-1) = -4$$

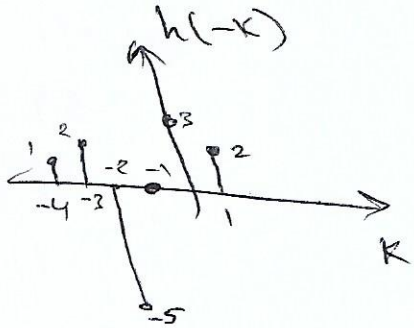


The end point of n is

max time index of input signal + max time index of impulse response

$$3 + 4 = 7$$

∴ The time index (n) of the output sequence will be $-4 \cdots 7$



$$n = [-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7)$$

$$y(-4) = \sum_{K} x(K)\, h(-4-K) = x(-3) \cdot h(-3) = 3 \cdot 2 = 6$$

$$= x(-3)\, h(-4-(-3)) = x(-3)\, h(-4+3)$$

$$= x(-3)\, h(-1) = 3 \cdot 2 = 6$$

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$$y(-3) = \sum_{-3}^{-3} x(k)\, h(-3-k) = x(-3)\cdot h(-3-(-3)) + x(-2)\cdot h(-3-(-2))$$

$$= x(-3)\cdot h(0) + x(-2)\cdot h(1)$$

$$= 3\cdot 3 + 11\cdot(-2) =$$

$$9 + 22 = 31$$

$$y(-2) = \sum_{-3}^{-1} x(k)\cdot h(-2-k) = x(-3)\cdot h(-2-(-3)) + x(-2)\cdot h(-2-(-2)) + x(-1)\cdot h(-2-(-1))$$

$$= x(-3)\,h(1) + x(-2)\cdot h(0) + x(-1)\,h(-1)$$

$$= 3\cdot 0 + 11\cdot 3 + 7\cdot 2 = 0 + 33 + 14 = \underline{47}$$

$y(-1) = 6$

$y(0) = -51$

$y(1) = -5$

$y(2) = 41$

$y(3) = 18$

$y(4) = -22$

$y(5) = -3$

$y(6) = 8$

$y(7) = 2$

$\therefore y(n) = [6, 31, 47, 6, -51, -5, 41, 18, -22, -3, 8, 2]$