Karnaugh Map Method :-

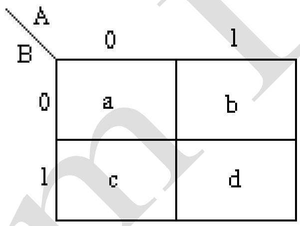
variable map examples

A Karnaugh map provides a pictorial method of grouping together expressions with common factors and therefore eliminating unwanted variables. The Karnaugh map can also be described as a special arrangement of a truth table.

The diagram below illustrates the correspondence between the Karnaugh map and the truth table for the general case of a two variable problem.

A	B	F
0	0	a
0	1	b
1	0	c
1	1	d

Truth Table.



F.

The values inside the squares are copied from the output column of the truth table, therefore there is one square in the map for every row in the truth table. Around the edge of the Karnaugh map are the values of the two input variable. A is along the top and B is down the left hand side. The diagram below explains this:

A	B	F
0	0	0
0	1	1
1	0	1 ←
1	1	1

Truth Table.

B \ A	0	1
0	0	1 ←
1	1	1

F.

The values around the edge of the map can be thought of as coordinates. So as an example, the square on the top right hand corner of the map in the above diagram has coordinates A=1 and B=0. This square corresponds to the row in the truth table where A=1 and B=0 and F=1. Note that the value in the F column represents a particular function to which the Karnaugh map corresponds.

Karnaugh Map (K-Map) Format :-

The fig. below shows three examples of K-maps for two, three, and four variables together with the corresponding truth tables :

B	A	X
0	0	1
0	1	0
1	0	0
1	1	1

	$\bar{B}$	B
$\bar{A}$	1	0
A	0	1

$X = \bar{A}\bar{B} + AB$

C	B	A	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	0
$\bar{A}B$	1	1
AB	0	0
$A\bar{B}$	1	0

$$X = \bar{A}B + \bar{B}\bar{C}$$

D	C	B	A	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	1	0
$A\bar{B}$	1	0	1	1

$$X = A\bar{B}\bar{D} + ACD$$

K-map examples :-

1)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	0	0	0	0

$$X = B$$

2)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	1	1	1

$$X = \bar{B}$$

3)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	0
$\bar{A}B$	1	1	0	0
AB	1	1	0	0
$A\bar{B}$	1	1	0	0

$$X = \bar{C}$$

4)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	1	0	0	1
AB	1	0	0	1
$A\bar{B}$	1	0	0	1

$$X = \bar{D}$$

5)

D (over $\bar{C}D$, CD) C (over CD, $C\bar{D}$)

B (rows 2–3) A (rows 3–4)

$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
0	0	0	1
0	1	1	0
0	1	1	0
0	0	1	0

$$X = BD + ACD + \bar{A}\bar{B}C\bar{D}$$

6)

$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
0	0	1	0
1	1	1	1
1	1	0	0
0	0	0	0

B, A

$$X = B\bar{C} + \bar{A}B + \bar{A}CD$$

7)

D, C

$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
0	1	0	0
0	1	1	1
1	1	1	0
0	0	1	0

B, A

$$X = \bar{A}BC + \bar{A}\bar{C}D + AB\bar{C} + ACD$$

Don't care :-

Some logic circuits can be designed so that there are certain input conditions for which there are no specified output levels, usually because these input conditions will never occur. In other words there will be certain combinations of input levels where we "don't care" weather the output is HIGH or LOW. This is illustrated in the following truth table.

C	B	A	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	X
1	0	0	X
1	0	1	1
1	1	0	1
1	1	1	1

Don't care

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	x
$\bar{A}B$	0	1
AB	X	1
$A\bar{B}$	0	1

$$X = C$$

3- Exclusive-OR (EX-OR) and Exclusive-NOR(EX-NOR) :-

Ex-OR:-

B	A	X	
0	0	0	
0	1	1	
1	0	1	
1	1	0	$X = \bar{A}B + A\bar{B} = A \oplus B$

Ex-NOR:-

B	A	X	
0	0	1	
0	1	0	
1	0	0	
1	1	1	$X = \bar{A}\bar{B} + AB = \overline{A \oplus B}$