Sum-of-products and the product-of-sums forms :-

Boolean Algebra :-

Boolean algebra differs in a major way from ordinary algebra in that Boolean constants and variables are allowed to have only two possible values, 0 or 1. Therefore, Boolean algebra is relatively easy to work with as compared to ordinary algebra.

Sum-of-Products Form

What does the sum-of-products form mean? First let us review products in Boolean algebra. A product of two or more variable or their complements is simply the AND function of these variables. The product of two variables can be expressed as AB, the product of three variables as ABC, the product of four variables as ABCD. Recall that a sum in Boolean algebra is the same as OR function, so a sum-of-products expression is two or more AND functions ORed together. For instance , AB+CD is a sum-of-products expression.

AB+BCD

ABC+DEF

A sum-of products form can also contain a term with a single variable, such as A+BCD+EFG

Product-of-sums forms.

The product-of-sums form can be thought of as the dual of the sum-of-products. It is , in terms of logic functions, the AND of two or more OR functions. For instance,(A+B)(B+C) is a product-of-sum expression

(A+B)(B+C+D)

(A+B+C)(D+E+F)

A product-of-sums expression can also contain a single variable term such as A(B+C+D)(E+F+G).

Boolean Theorems :-

1) $A \cdot 0 = 0$
2) $A \cdot 1 = A$
3) $A \cdot A = A$
4) $A \cdot \bar{A} = 0$
5) $A + 0 = A$
6) $A + 1 = 1$
7) $A + A = A$
8) $A + \bar{A} = 1$
9) $A + B = B + A$
10) $A \cdot B = B \cdot A$
11) $A + (B+C) = (A+B)+C$
    $= A + B + C$
12) $(A+B).(C+D) =$
    $A.C + A.D + B.C + BD$
13) $A + A.B = A$
14) $A + \bar{A} . B = A + B$

Example:

Simplify the expression :

$$y = A\bar{B}D + A\bar{B}\bar{D}$$

<u>solution:</u>

$$y = A\bar{B}(D + \bar{D}) = A\bar{B}.1 = A\bar{B}$$

Example:

Simplify the expression :

$$z = (\bar{A} + B)(A + B)$$

<u>solution:</u>

$$z = \bar{A}.A + \bar{A}.B + B.A + B.B$$
$$= 0 + B(\bar{A} + A) + B$$
$$= 0 + B + B = B$$
$$\therefore z = B$$