Numbers Systems :-

1. The types of number system

A-Decimal Number System :-

This system is composed of 10 numbers or symbols, these 10 symbols are:

0 1 2 3 4 5 6 7 8 9

These symbols are called digits.

The decimal system, also called base 10 system, because it has 10 digits which is a naturally result of the fact that man has 10 fingers.

B- Binary Number System

In this system there are only two symbols or possible digit values , 0 or 1 . Even so , this base-2 system.

C- Octal Number System

This system is composed of 8 numbers or symbols:

0 1 2 3 4 5 6 7

This is a base -8 system.

D- Hexa- Decimal System

This system is composed of 16 numbers or symbols (digit):

0 1 2 3 4 5 6 7 8 9 A B C D E F

It is a base – 16 systems

2. Representation of numbers

1) Decimal :-

$(124)_{10} = 4 \times 10^0 + 2 \times 10^1 + 1 \times 10^2$

$(252.512)_{10} = 2 \times 10^0 + 5 \times 10^1 + 2 \times 10^2 + 5 \times 10^{-1} + 1 \times 10^{-2} + 2 \times 10^{-3}$

2)Binary :-

$(1011101)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 + 0 \times 2^5 + 1 \times 2^6$

$= (93)_{10}$

$(101.11)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$=(5.75)_{10}$

3) Octal :-

$(537)_8 = 7 \times 8^0 + 3 \times 8^1 + 5 \times 8^2$

$=(351)_{10}$

4) Hexa- Decimal :-

$(A01B)_{16} = 11 \times 16^0 + 1 \times 16^1 + 0 \times 16^2 + 10 \times 16^3$

$=(40987)_{10}$

1. Convert between the types of numbers systems

any base-to-decimal conversion :-

just use the definition given above.

Decimal-to-binary :-

divide decimal value by 2 (the base) until the value is 0

example: convert the following decimal numbers to the equivalent binary numbers (36 , 39.5).

36/2 = 18 r =0
18/2 = 9 r =0
9/2 = 4 r =1
4/2 = 2 r =0
2/2 = 1 r =0
1/2 = 0 r =1

36 (base 10) = 100100

39/2 = 19 r =1
19/2 = 9 r = 1
9/2 = 4 r = 1
4/2 = 2 r = 0
2/2 = 1 r = 0
1/2 = 0 r = 1

39.5 (base 10) == 100111
10

0.5 x 2 = 1.0
0 x 2 = 0

The binary equivalent of $(39.5)_{10}$ is $(100111.10)_2$

Decimal-to-Octal :-

A decimal integer can be converted to octal by using the same repeated-division method that we used in the decimal-to-binary conversion, but with a division of 8 instead of 2.

Example: convert the following decimal number to equivalent octal number $(266)_{10}$ & $(20.75)_{10}$

266/8 = 33 r =2
33/8 = 4 r =1
4/8 = 0 r =4

$(266)_{10}$ = 412

20/8 = 2 r =4 0.75 x 8 = 6.0
2/8 = 0 r =2

The equivalent octal number is $(24.6)_8$

Octal-to-binary :-

The conversion from octal to binary is performed by converting each octal digit to its 3-bit binary equivalent. The eight possible digits are converted as indicated in the following table:

Octal digit	0	1	2	3	4	5	6	7
Binary digit	000	001	010	011	100	101	110	111

Example: convert the following octal number to it's equivalent binary number $(472)_8$

4 7 2
100 111 010

The equivalent binary number is $(100111010)_2$

Binary-to-octal :-

1. group into 3's starting at least significant symbol (if the number of bits is not evenly divisible by 3, then add 0's at the most significant end)
2. write 1 octal digit for each group

example:

100	010	111	(binary)
4	2	7	(octal)

10	101	110	(binary)
2	5	6	(octal)

example:-

convert $(177)_{10}$ to its 8-bit binary equivalent by first converting to octal.

Solution:-

177/8 = 22 + reminder of 1
22/8 = 2 + reminder of 6
2/8 = 0 + reminder of 2

$(177)_{10} = (261)_8$

2	6	1
010	110	001

$(177)_{10} = (261)_8 = (010110001)_2$

Tutorials:-

1- Convert $(641)_8$ to decimal (Ans. 369).
2- Convert $(146)_{10}$ to octal then from octal to binary (Ans. 222 and 010010010).
3- Convert $(10011101)_2$ to octal (Ans. 235).
4- Write the next three numbers in this octal counting sequence: 624, 625, 626, ….., ….., ……
5- Convert $(975)_{10}$ to binary by first converting to octal (Ans. 1111001111).
6- Convert binary 1010111011 to decimal by first converting to octal (Ans. 699).

Decimal-to-hex :-

This conversion can be done using repeated division by 16.

Example: convert $(423)_{10}$ to hex.

423/16 = 26 r =7

26/16 = 1 r = 10

1/16 = 0 r =1

1A7

Hex-to-binary :-

Each hex digit is converted to it's 4-bit binary equivalent as show in the table below :

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

example:

3 9 C 8

0011 1001 1100 1000

Binary-to-hex :-

This conversion is just the reverse of the hexa-to-binary conversion process.

example:

1001	1110	0111	0000
9	E	7	0

1	1111	1010	0011
1	F	A	3

Tutorials:-

1- Convert $(24CE)_{16}$ to decimal (Ans. 9422).
2- Convert $(3117)_{10}$ to hex, then from hex to binary (Ans. C2D and 110000101101)
3- Convert $(1001011110110101)_2$ to hex (Ans 97B5).
4- Write the next four numbers in this hex counting sequence: E9A, E9B, E9C, E9D, ….., ……., ……, …….
5- Convert $(3527)_8$ to hex (Ans. $(757)_{16}$).

Binary-coded-decimal code :-

If each digit of a decimal number is represented by its binary equivalent, this produces a code called binary-coded-decimal (BCD). Since a decimal digit can be as large as 9, 4-bits are required to code each digit. The table below shows each decimal digit and its binary equivalent.

Decimal	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

example: The BCD of decimal 874

8	7	4
1000	0111	0100

Excess-3-code :-

It is performed in the same manner as BCD except that 3 is added to each decimal digit before encoding it in binary. The following table shows this code.

Decimal	0	1	2	3	4	5	6	7	8	9
Ex-3-code	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100

Gray code :-

The Gray code belongs to a class of codes called minimum-change codes, in which only from one step to the next. The following table shows this code.

Decimal	0	1	2	3	4	5	6	7	8	9
Gray-code	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101