Lecture six

Objective of Lecture:

- Intersymbole Interference (ISI)
- Pulse Shaping to reduce ISI
- Equalizer
- Adaptive Equalizer, Matched Filter.

Intersymbol Interference (ISI):

ISI arises because dispersive nature of the communication channel, thus the errors are introduced in the detected data at the receiver. The binary data can be transmitted in baseband or passband. One of the baseband system of digital data is PAM which is have only two amplitude level corresponding to binary "1" and "0". Successive binary digits can combined into symbol. Line codes generate discrete PAM signals which transmitted in baseband form (without any modulation) over the channel. Fig. 5 shows the block diagram of such baseband transmission system.

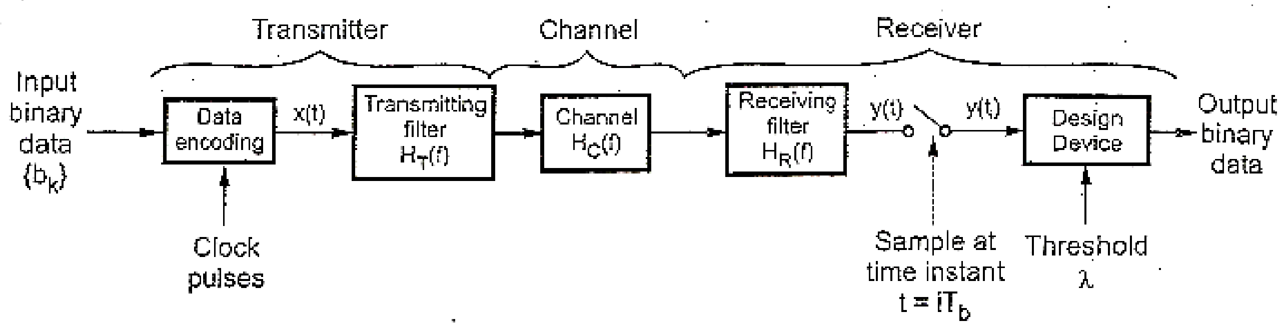


Figure 15

The binary data *(b_k)* is applied to the data encoder which generate the pulse waveform *x(t)* represented mathematically by:

$$x(t) = \sum_{k=-\infty}^{\infty} A_k\, g(t - kT_b)$$

T_b is the duration of each input binary bit.

$g(t)$ is the shaping pulse.

And $$A_k = \begin{cases} +a & if \ \ b_k = 1 \\ -a & if \ \ b_k = 0 \end{cases}$$

The signal *x(t)* is then passed through the transmitting filter which combine all the necessary transmitting circuits and systems with transfer function of $H_T(f)$. The signal is then passed through the channel having transfer function $H_C(f)$. The channel delivers the signal to the receiving filter which combine all the necessary receiving circuits and systems with transfer function of $H_R(f)$. The output of the receiving filter is *y(t)* is a noisy replica of the transmitted signal *x(t)*. The signal *y(t)* is then sampled synchronously with clock pulse at the transmitter having sampling instants $t = iT_b$. The sampled signal $y(t_i)$ is then given to decision device which compare the input signal with threshold "λ":

If $y(t_i) > \lambda$ select symbol '1'

If $y(t_i) \leq \lambda$ select symbol '0'

ISI Problem: Consider the output *y(t)* of the receiving filter represented by:

$$y(t) = \mu \sum_{k=-\infty}^{\infty} A_k \, p(t - kT_b)$$

Here μ is scaling factor and *P(t)* is the different from that of *g(t)*

We have $t = iT_b$ then:

$$y(t) = \mu \sum_{k=-\infty}^{\infty} A_k \, p(iT_b - kT_b)$$

$$y(t) = \mu \sum_{k=-\infty}^{\infty} A_k \, p(i - k)T_b$$

Let us rearrange above equation:

$$y(t) = \mu \, A_i \, p(0) + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k \, p(i - k)T_b$$

$$and \ i = 0, \mp 1, \mp 2, \mp 3, \dots \dots \dots.$$

If ISI is absent then the second term of above equation will not be present, and *p(t)* is normalized such that $p(0) = 0$ yelds: $y(t) = \mu A_i$. At $t = iT_b$, the correct bit is A_i. Observe that it is decoded correctly in absence of ISI, but it is not possible to eliminate the second term totally. The ISI can be reduced by proper design of transmitted filter $H_T(f)$, receive filter $H_R(f)$, and channel filter $H_C(f)$.

Equalization:

When the signal is passed through the channel, distortion is introduced in terms of (i) amplitude and (ii) delay. This distortion creates the problems of ISI. The detection of the signal also becomes difficult. This distortion can be compensated with the help of *equalizers*. Equalizers are basically filters, which correct the channel distortion. Fig.4 shows channel and equalizer for correction of distortion.

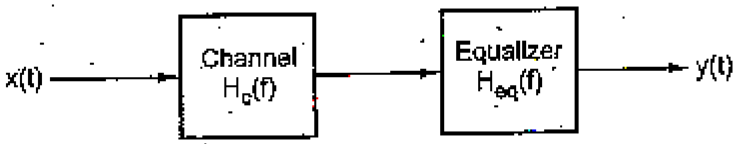


Figure 4

The transfer function of distortionless system is given as,

$$H(f) = Ke^{-j2\pi f t_0}$$

The cascade connection of channel + equalizer shown in above figure will be distortionless if,

$$H_c(f) \cdot H_{eq}(f) = Ke^{-j2\pi f t_0}$$

Hence transfer function of the equalizer will be,

$$H_{eq}(f) = \frac{Ke^{-j2\pi f t_0}}{H_c(f)}$$

The equation is difficult to realize directly, but approximations are available. It can be implemented with the help of tapped delay line filters.

Fig. 5 shows tapped delay line filter. The output of such filter is given as:

$$y(nT) = \sum_{i=0}^{M-1} w_i x\,(nT - iT)$$

Here w_i is the weight of i^{th} tap, M is the total number of taps and T is the symbol duration of the signal.

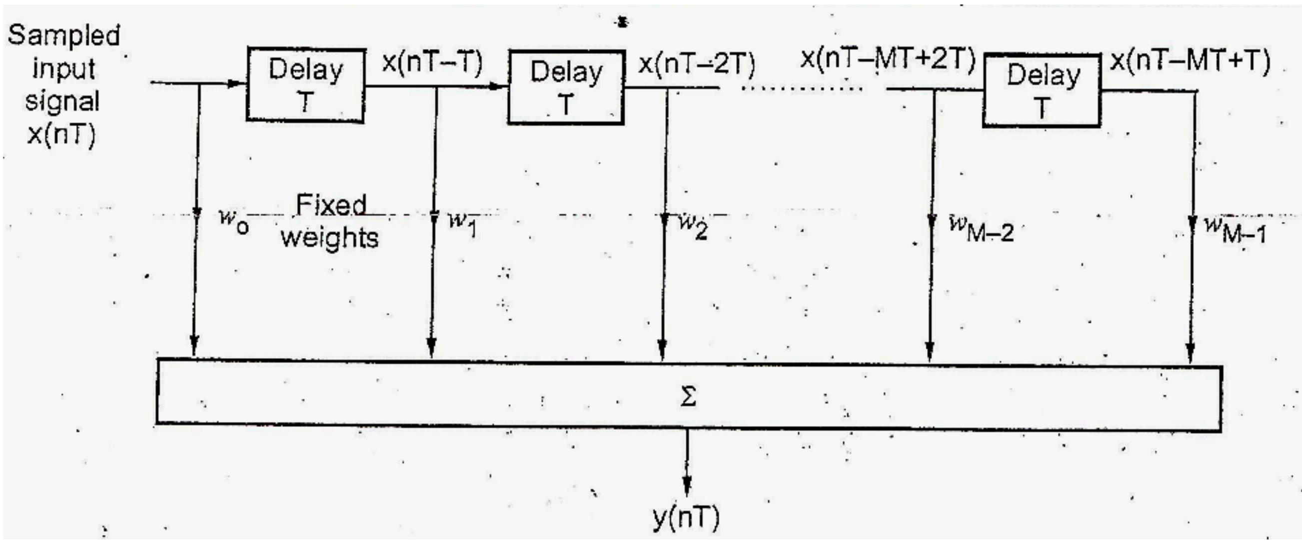


Figure 5

The weights are basically filter coefficients. This filter approximates the equalizer transfer function of above equation. The approximation will be more accurate if we use more tape in filter.

Adaptive Equalizer:

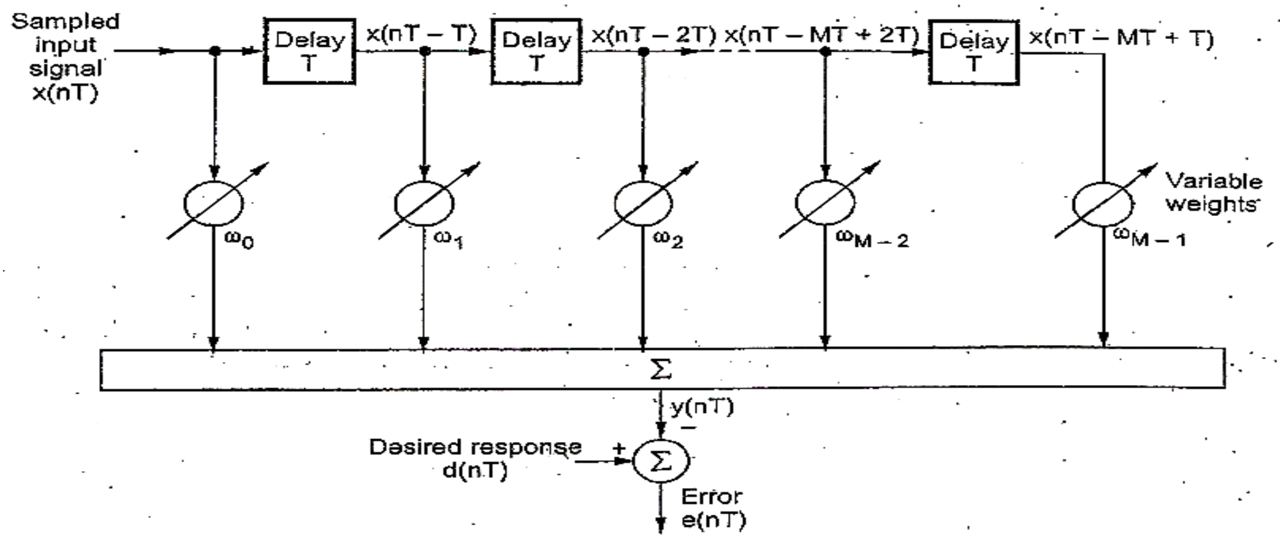
The transmission characteristic of the channel keep in changing. Hence adaptive equalization is used. In adaptive equalization, the filters adapt themselves to the dispersive effects of the channel. That is the coefficients of the filters are changed continuously according to the received data.

The adaptive equalizer is a tapped delay-line filter. It consist of set of delay elements and variable multipliers. The sequence $x(nT)$ is applied to the input of adaptive filter. The output $y(nT)$ of the adaptive filter will be,

$$y(nT) = \sum_{i=0}^{M} w_i x\,(nT - iT)$$

The weights w_i on the taps are basically adaptive filter coefficients. A known sequence $\{d(nT)\}$ is transmitted first. This sequence is known to the receiver. The error between two sequences is calculated:

$$e(nT) = d(nT) - y(nT), \quad n = 0, 1, 2, \ldots\ldots N-1$$



The weights of the filter are changed recursively such that error $e(nT)$ is minimized. There are standard algorithms to change weights of filter recursively. One of important algorithm is Least Mean Square (LMS) Algorithm.

The tap weights are adapted by this algorithm as follows:

$$\acute{w}_i(nT + T) = \acute{w}_i(nT) + \mu e(nT)x(nT - iT) \quad i = 0, 1, \ldots\ldots M-1$$

$\acute{w}_i(nT)$ is the present estimate for tap i at time nT,

$\acute{w}_i(nT + T)$ is the updated estimate for tap i at time nT,

μ is the adaption constant, $x(nT - iT)$ is the filter input

and $e(nT)$ is the error signal.

In this algorithm initial tap weights are assumed zero.

The Matched Filter:

A matched filter is a linear filter designed to provide the maximum signal to noise ratio at its output for a given transmitted symbol waveform. Consider that a known signal *s(t)* plus AWGN *n(t)* is the input to a linear time- invariant (receiving filter followed by a sampler. The ratio of instantaneous signal power to average noise power, $(\frac{S}{N})_T$, at time $t = T$, out of sampler is

$$(\frac{S}{N})_T = \frac{a_i^2}{\sigma_0^2}$$

We can express the signal $a_i(t)$ at the filter output in terms of filter transfer function:

$$a_i(t) = \int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi ft}df$$

Where $S(f)$ is the Fourier transform of input signal, $s(t)$. If the two sided power spectral density of the input is $\frac{N_0}{2}$ watts/hertz, then we can express the output noise power as:

$$\sigma_0^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df$$

Substitute in the first equation:

$$(\frac{S}{N})_T = \frac{\left|\int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi ft}df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

Using Schwarz's inequality:

$$\left|\int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi ft}df\right|^2 \leq \int_{-\infty}^{\infty}|H(f)|^2 df \int_{-\infty}^{\infty}|S(f)|^2 df \qquad \text{yields}$$

$$(\frac{S}{N})_T = \frac{2}{N_0}\int_{-\infty}^{\infty}|S(f)|^2 df$$

$$or \qquad max\ (\frac{S}{N})_T = \frac{2E}{N_0}$$

Where the energy *E* of the input signal *s(t)*is

$$E = \int_{-\infty}^{\infty}|S(f)|^2 df$$

Thus the maximum output $(\frac{S}{N})_T$ depends on the input signal energy and the and the power spectral density of the noise, not on the particular shape of the waveform that used.

$max\ (\frac{S}{N})_T$ holds only if the optimum filter transfer function $H_0(f)$ is employed:

$$H(f) = H_0(f) = kS^*(f)e^{j2\pi ft}$$

$$\text{Or} \qquad h(t) = IFT\{\ kS^*(f)e^{-j2\pi ft}\}$$

Since s(t) is a real valued signal,

$$h(t) = \begin{cases} ks(T-t) & 0 \leq t \leq T \\ 0 & elsewhere \end{cases}$$

Thus, the impulse response of a filter that produces the maximum output signal to noise ratio is the mirror image of the message signal $s(t)$ delayed by the symbol time duration T.