Lecture Four

Objective of Lecture:

- Pulse Code Modulation PCM)
- Noise Consideration in PCM
- Limitation and Modifications of PCM
- Differential PCM (DPCM).

Introduction:

Analog waveform or signals are sampled into pulses. In digital pulses modulation methods, the analog amplitude pulses are converted to digital form. Thus each sample of the message signal is represented in binary (1, 0) format.

2.1 Pulse Code Modulation:

The PCM technique samples the input signal x(t) at frequency $f_s \geq 2W$. This sampled pulse is then digitized by the analog to digital converter as shown in Fig. 2-1.

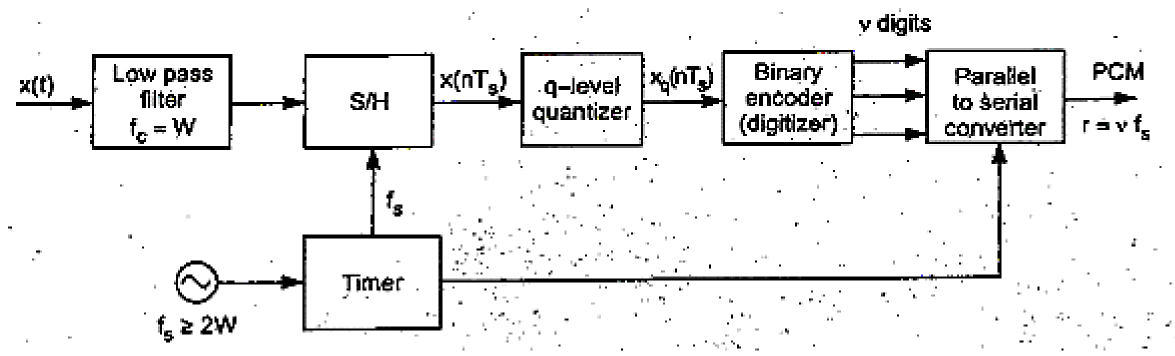

Figure 2-1 the block diagram of PCM

The x(t) is bandlimited to W by LPF. The sample and hold circuit then samples this signal at the rate above of Nyquist rate $f_s \geq W$. The sampled signal $x(nT_s)$is discrete in time and continuous in amplitude. The quantizer is convert it to q discrete level by rounding each sample to fixed digital level with minimum error (quantization error). The input to the quantizer $x(nT_s)$ (for example) can take any values between $(-4\delta\ to + 4\delta)$, the output of quantizer $(x_q(nT_s)$ are available at $\pm\frac{\delta}{2}, \pm\frac{3\delta}{2}, \pm\frac{5\delta}{2}\ and\ \pm\frac{7\delta}{2}$ as shown in Fig. 2-2. Thus the maximum quantization error is $\pm\frac{\delta}{2}$.

Transmission bandwidth in PCM:

Each quantized sample can be represent by v digits: $q = 2^v$, where q is the total number of digital levels.

The number of sample is f_s, and each sample represent by v bits then:

Signaling rate of PCM: $r = v \times f_s$

And we have $f_s \geq 2W$.

The bandwidth of PCM given half signaling rate: $B_r \geq \frac{1}{2} r$

$B_r \geq \frac{1}{2} v f_s$

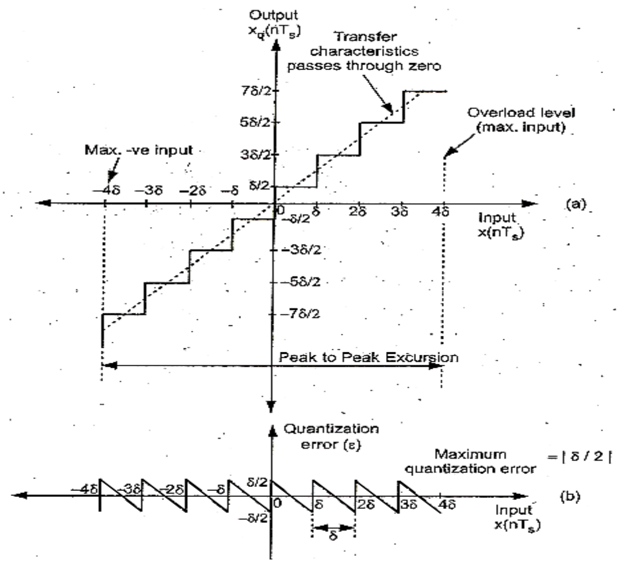

Figure 2-2:(a) Transfer characteristic of a quantizer

(b) Variation of quantizer error

Since $f_s \geq 2W$

$\therefore\ B_r \geq vW$

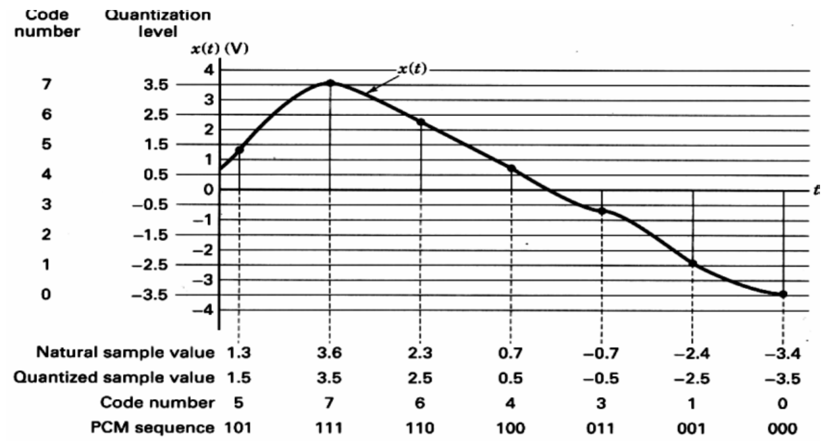

Figure 2-3: quantizing of sample value

2-2 PCM Receiver:

Figure 2-4 shows the block diagram of PCM receiver. The regenerator is to reshapes the pulse and removes the noise. The signal is then converted into parallel digital words for each sample.

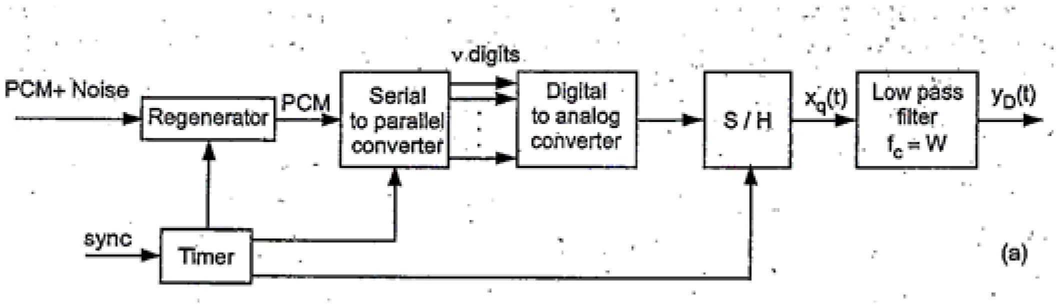

Figure 2-4 PCM receiver

The digital word is converted to its analog value $x_q(t)$ along with sample and hold (S/H), then passed through lowpass reconstruction filter to get $y_D(t)$. There is quantization error between reconstructed signal $x(kT_s)$ and original signal $x(t)$ as shown in figure 2-5. This can reduced by increasing bits 'v', but this increases the bandwidth.

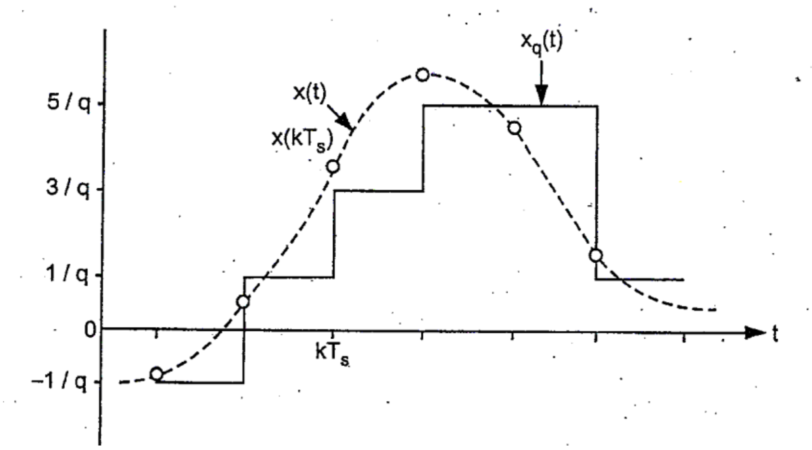

Figure 2-5: Reconstructed waveform

2-3 Quantization Noise in PCM:

The quantization error is expressed as:

$$\varepsilon = x_q(nT_s) + x(nT_s)$$

The range of amplitude of input $x(nT_s)$ is $-x_{max}\ to + x_{max}$ and it is mapped into q levels. So that total amplitude range $2x_{max}$ is divided into q levels with step size δ.

$$\delta = \frac{2x_{max}}{q}$$

We have the maximum quantization error is $\pm\frac{\delta}{2}$, or $\varepsilon_{max} = \left|\frac{\delta}{2}\right|$

The mean square value of quantization error is:

$$E(\varepsilon^2) = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \varepsilon^2 f_\varepsilon(\varepsilon) d\varepsilon = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \varepsilon^2 \frac{1}{\delta} d\varepsilon = \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\frac{\delta}{2}}^{\frac{\delta}{2}}$$

$$E(\varepsilon^2) = \frac{1}{\delta}\left[\frac{\left(\frac{1}{\delta}\right)^3}{3} + \frac{\left(\frac{1}{\delta}\right)^3}{3}\right] = \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right] = \frac{\delta^2}{12}$$

The noise power= $\frac{V_{noise}^2}{R}$

Assume R=1, then the noise power (normalized)= $\frac{V_{noise}^2}{1}$

$$E(\varepsilon^2) = \frac{\delta^2/12}{1} = \frac{\delta^2}{12}$$

The maximum signal power to quantization noise ratio:

$$\frac{S}{N} = \frac{Normalized\ signal\ power}{\delta^2/12}$$

We have $q = 2^v$, so:

$$\delta = \frac{2x_{max}}{q} = \frac{2x_{max}}{2^v}$$

Substituting in above $\frac{S}{N}$ equation:

$$\frac{S}{N} = \frac{Normalized\ signal\ power}{\frac{\left(\frac{2x_{max}}{2^v}\right)^2}{12}}$$

Let normalized signal power as P

$$\frac{S}{N} = \frac{P}{\left(\frac{2x_{max}}{2^v}\right)^2/12} = \frac{3P}{(x_{max}^2)} \times 2^{2v}$$

This equation shows that the signal to noise power ratio of quantizer increases exponentially with increasing bits per sample. For normalized x_{max}

$$\frac{S}{N} = 3 \times 2^{2v} \times P$$

$$\left(\frac{S}{N}\right) dB = 10log_{10}\left(\frac{S}{N}\right) dB = 10log_{10}(3 \times 2^{2v}) = (4.8 + 6v)dB$$

For normalized values of power the destination signal power ‘P’ is less than 1

So that

$$\left(\frac{S}{N}\right) dB \leq (4.8 + 6v)dB$$

Example 1:

A television signal with bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantizatin levels is 512. Calculate:

i- Code word length

ii- Transmission bandwidth

iii- Final bit rate

iv- Output signal to quantization noise ratio

Solution:

i- $q = 2^v \rightarrow 512 = 2^v$

$$log512 = v \times log2 = 9\ bits$$

Thus the code word length is 9 bits

ii- $B_T \geq vW \rightarrow B_T \geq 9 \times 4.2 \times 10^6$

$$\therefore\ B_T \geq 37.8\ MHz$$

iii- The signaling rate $r = v \times f_s = v \times 2W = 9 \times 2 \times 4.2 \times 10^6$

$$r = 75.6\ Mbps$$

Also we have $B_T \geq \frac{1}{2}r \quad or\ B_T \geq 0.5 \times 75.6 = 37.8\ MHz$ which is same value obtained earlier

iv- $\left(\frac{S}{N}\right) dB = (4.8 + 6v)dB = 4.8 + 6 \times 9 = 58.8\ dB$

Example 2:

The bandwidth of signal input to the PCM is restricted to 4 kHz. The input varies from -3.8V to +3.8 V and has the average power of 30 mW. The required signal to noise ratio is 20 dB.

i- Calculate the number of bits required per sample.

ii- Outputs of 20 such PCM coder are time multiplexed. What is the mimum required transmission bandwidth for the multiplexed signal?

Solution:

$$\left(\frac{S}{N}\right)dB = 10log_{10}\left(\frac{S}{N}\right)dB = 20dB$$

$$\therefore \qquad \frac{S}{N} = 100$$

i- $$\frac{S}{N} = \frac{3P}{(x_{max}^2)} \times 2^{2v} \rightarrow 100 = \frac{3\times30\times10^{-3}\times2^{2v}}{(3.8)^2}$$

$$2^{2v} = \frac{1444}{0.06} = 24066.67$$

$$2vlog2 = \log(24066.67)$$

$$v \cong 7\ bits$$

ii- $B_T \geq vW$ and for 20 multiplexed signals

$$B_T \geq 20 \times 7 \times 4\ kHz \geq 840\ kHz$$

And signaling rate

$$r = 2B_T = 2 \times 840 = 1680\ kbps$$

Example 3:

The information in an analog signal voltage waveform is to be transmitted over a PCM system with an accuracy of $\mp 0.1\%$ (full scale). The analog signal has a bandwidth of 100Hz and an amplitude range of -10 to +10 volts. Determine:

i- The number of levels required for such accuracy.

ii- The code word length.

iii- The minimum bit rate required.

iv- The bandwidth required for PCM signal.

Solution:

i- The maximum quantization error should be $\mp 0.1\%$, so:

$$\varepsilon_{max} = \mp 0.001$$

But $\varepsilon_{max} = \left|\frac{\delta}{2}\right|$

$$\left|\frac{\delta}{2}\right| = 0.001 \rightarrow the\ step\ size\ \delta = 0.002$$

We have $\delta = \frac{2x_{max}}{q}, and\ |x_{max}| = 10\ volts$

Then $0.002 = \frac{2\times 10}{q} \rightarrow q = 10000$

So that the number of levels are 10000.

ii- We have $q = 2^v \rightarrow 10000 = 2^v \rightarrow log10000 = v\ log2$

$$\therefore the\ code\ word\ length \quad v = 13.288 \cong 14\ bits$$

iii- We have the bit rat $r = vf_s = v \times 2 \times W = 14 \times 2 \times 100 = 2800\ bps$

iv- The bandwidth required $B_T \geq \frac{1}{2}r \geq \frac{1}{2} \times 2800 = 1400\ Hz$

H.W:

Q1/ The information in an analog waveform with maximum frequency $f_m = 3\ kHz$ is to be transmitted over 16- levels PCM system. The quantization distortion is specified not exceed 1% of peak to peak analog signal.

i- What is the number of bits per sample that should be used in this PCM?

ii- What is minimum bit transmission rate?

Q2 / A signal of bandwidth 3.5 kHz is sampled, quantized and coded by PCM system. The code signal is then transmitted over a transmission channel of supporting a transmission rate of 50 kbps. Calculate the maximum signal to noise that can obtained by this system. The input signal has peak to peak value of 4 volts and rms value of 0.2 V.

Q3 / Consider an audio signal comprised of the sinusoidal term $s(t) = 3\cos(500\pi t)$.

i- Find the number of quantization level with an accuracy of 1%.

ii- Determine the signaling rate.

iii- The bandwidth of transmission channel.

2-4 Advantages of PCM:

i- Effect of channel noise and interference is reduced.

ii- PCM permits regeneration of pulses along the transmission path. This reduces noise interference.

iii- The bandwidth and signal to noise ratio are related by exponential law.

iv- Multiplexing of various PCM signals is easily possible.

v- Encryption or decryption can easily incorporated for security purpose.

2-5 Limitation of PCM:

i- PCM system are complex compared to analog pulse modulation method.

ii- The channel bandwidth is also increased because of digital coding of analog pulses.

2-6 Modifications of PCM:

i- PCM can be modified to delta modulation. It is more simplified method of implementation.

ii- The PCM can be used in wideband communications channels to overcome the bandwidth problem.

iii- With the help of data comparison along with PCM, the redundancy can be removed and data rate can be reduced.

2-7 Differential Pulse Code Modulation (DPCM):

Any signal does not change fast, so that the value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with little difference as shown if figure 2-11.

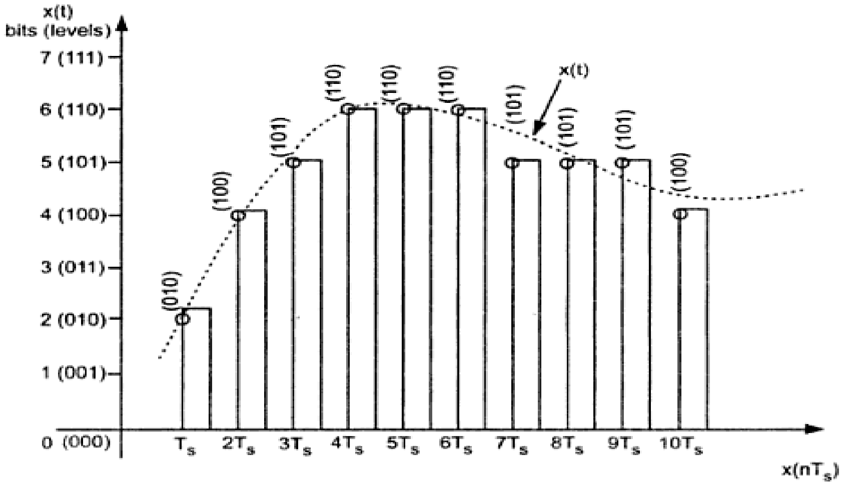

Figure 2-6 Redundant information of PCM

It can be seen from figure 2-11 the samples $4T_s, 5T_s, and\ 6T_s$ are encoded to the same value of (110). If this redundancy is reduced, the overall bit rate will decrease and the number of bits required for one sample will also be reduced. This is called Differential Pulse Code Modulation (DPCM).

DPCM works on the principle of prediction. The value of the present sample is predicted from the past samples as shown in figure 2-12.

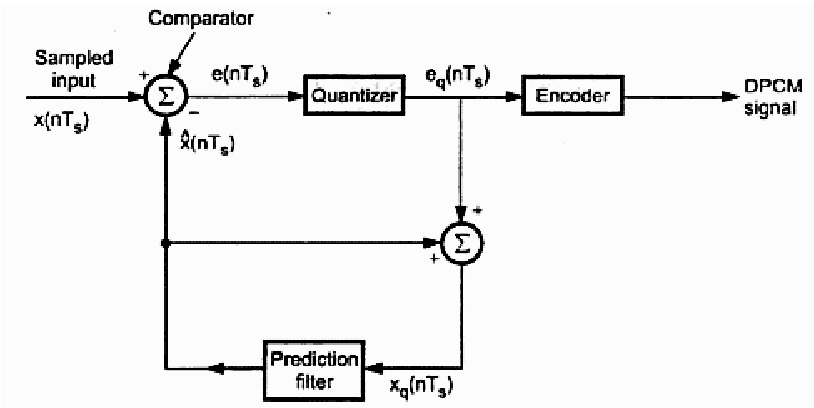

Figure 2-7 DPCM transmitter

The comparator fined the difference between the actual sample value $x(nT_s)$ and predicted signal $\hat{x}(nT_s)$ this is called error $e(nT_s)$:

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

This error will be quantized and encoded by small number of bits. Thus number of bits per sample are reduced in DPCM. The quantization error can be written as:

$$e_q(nT_s) = e(nT_s) + q(nT_s)$$

The prediction filter input is:

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$$

Substituting by $e_q(nT_s)$ yields

$$x_q(nT_s)=\hat{x}(nT_s) + e(nT_s) + q(nT_s)$$

We have

$$x(nT_s) = e(nT_s) + \hat{x}(nT_s)$$

From the last equations;

$$x_q(nT_s)=x(nT_s) + q(nT_s)$$

To reconstruct the original signal at the receiver, the decoder first reconstructs the quantized error signal as shown in figure 2-13.

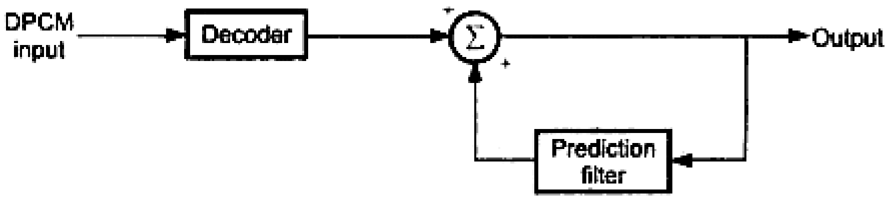

Figure 2-8 Reconstruction of DPCM

The quantized error signals are summed up with prediction filter output to give the quantized version of the original signal. The signal at the receiver differs from actual signal by quantization error $q(nT_s)$.