Forced (harmonically excited) single DoF vibration – undamped.

- Let the forcing function acting on the mass of an undamped SDOF system be:

$$F(t) = F_0 \cos(\omega t)$$

- The eqn. of motion reduces to:

$$m\ddot{x} + kx = F_0 \cos(\omega t)$$

- Where the homogeneous solution is:

$$x_h(t) = C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t)$$

$$where\ \omega_n = \sqrt{k/m}$$

- As the excitation is harmonic, the particular solution is also harmonic with the same frequency:

$$x_p(t) = X \cos(\omega t)$$

- Substituting $x_p(t)$ in the eqn. of motion and solving for X gives:

$$X = \frac{F_0}{k - m\omega^2}$$

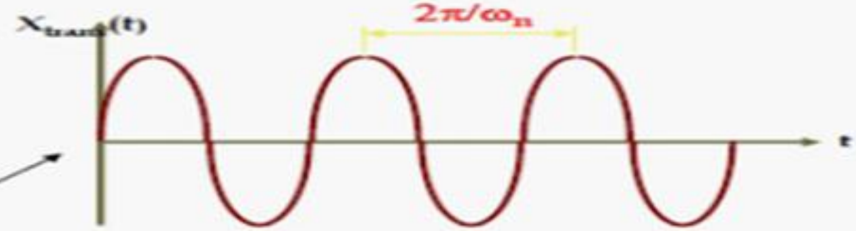
- The complete solution becomes $x(t) = x_h(t) + x_p(t)$

$$x(t) = x_h(t) + x_p(t) = C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t) + \frac{F_0}{k - m\omega^2} \cos(\omega t)$$

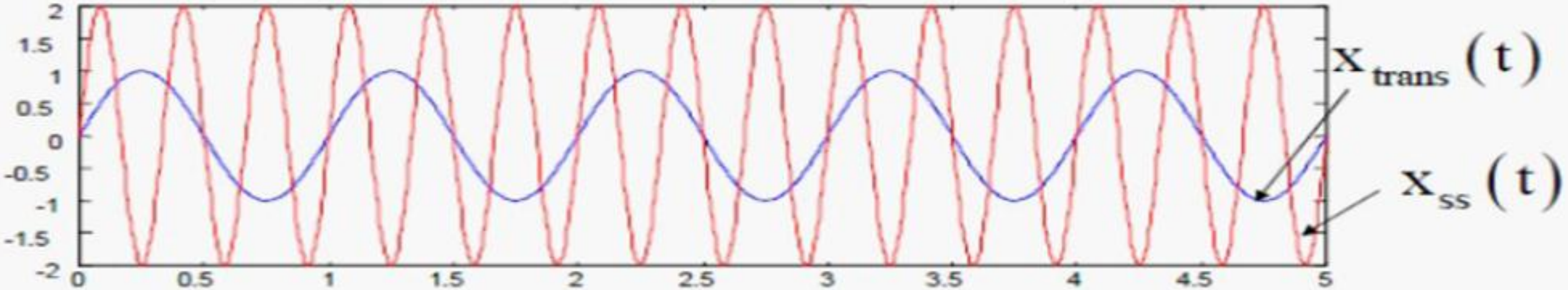
$$m\ddot{x} + kx = F_o \cos(\omega t)$$

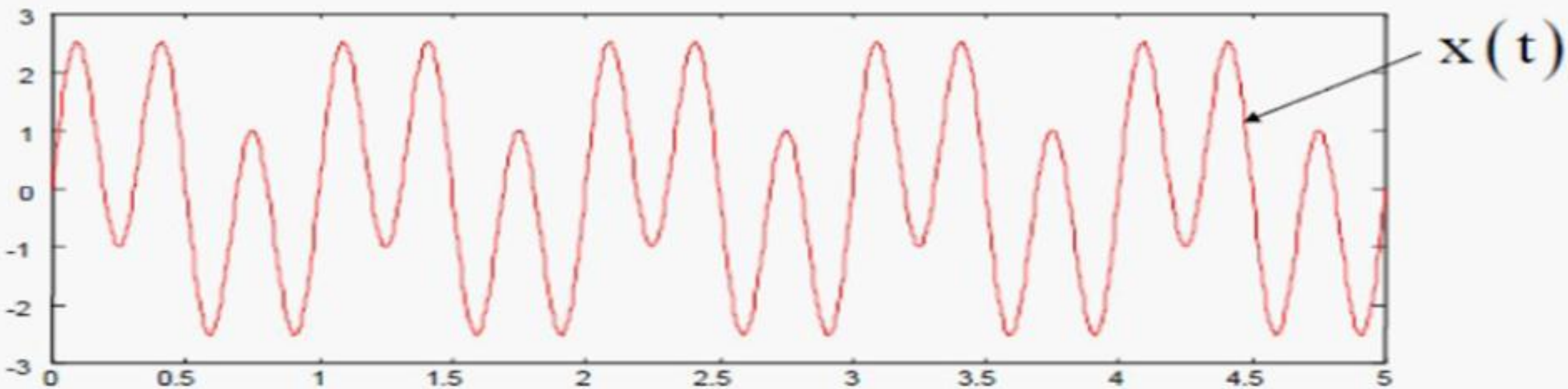
$$x(t) = x_h(t) + x_p(t)$$

$$\text{or } x(t) = x_{trans}(t) + x_{ss}(t)$$



$$x_{trans}(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$$

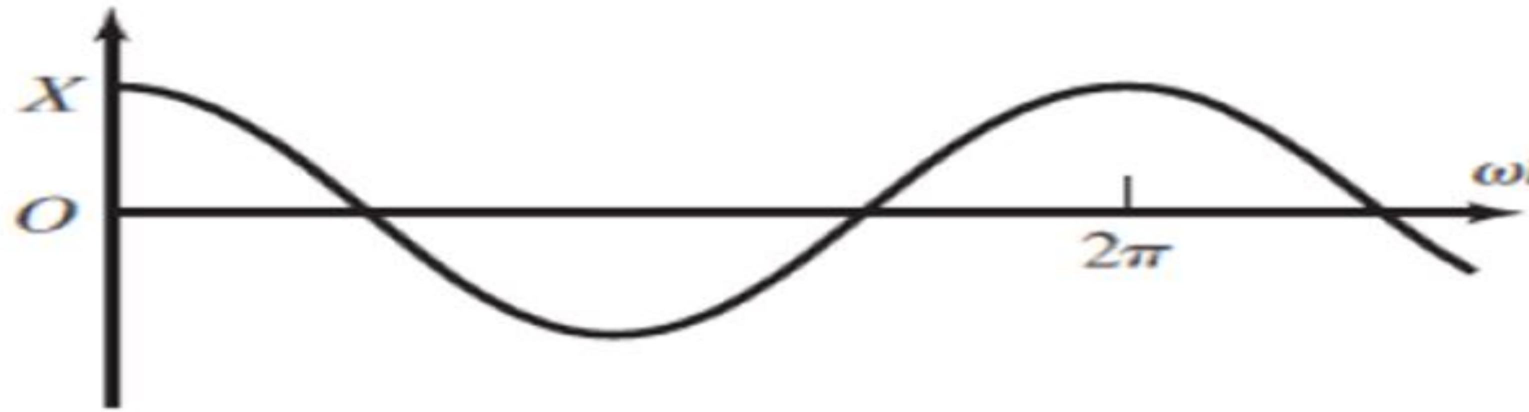




$0 < \omega / \omega_n < 1.$

$F(t) = F_0 \cos \omega t$

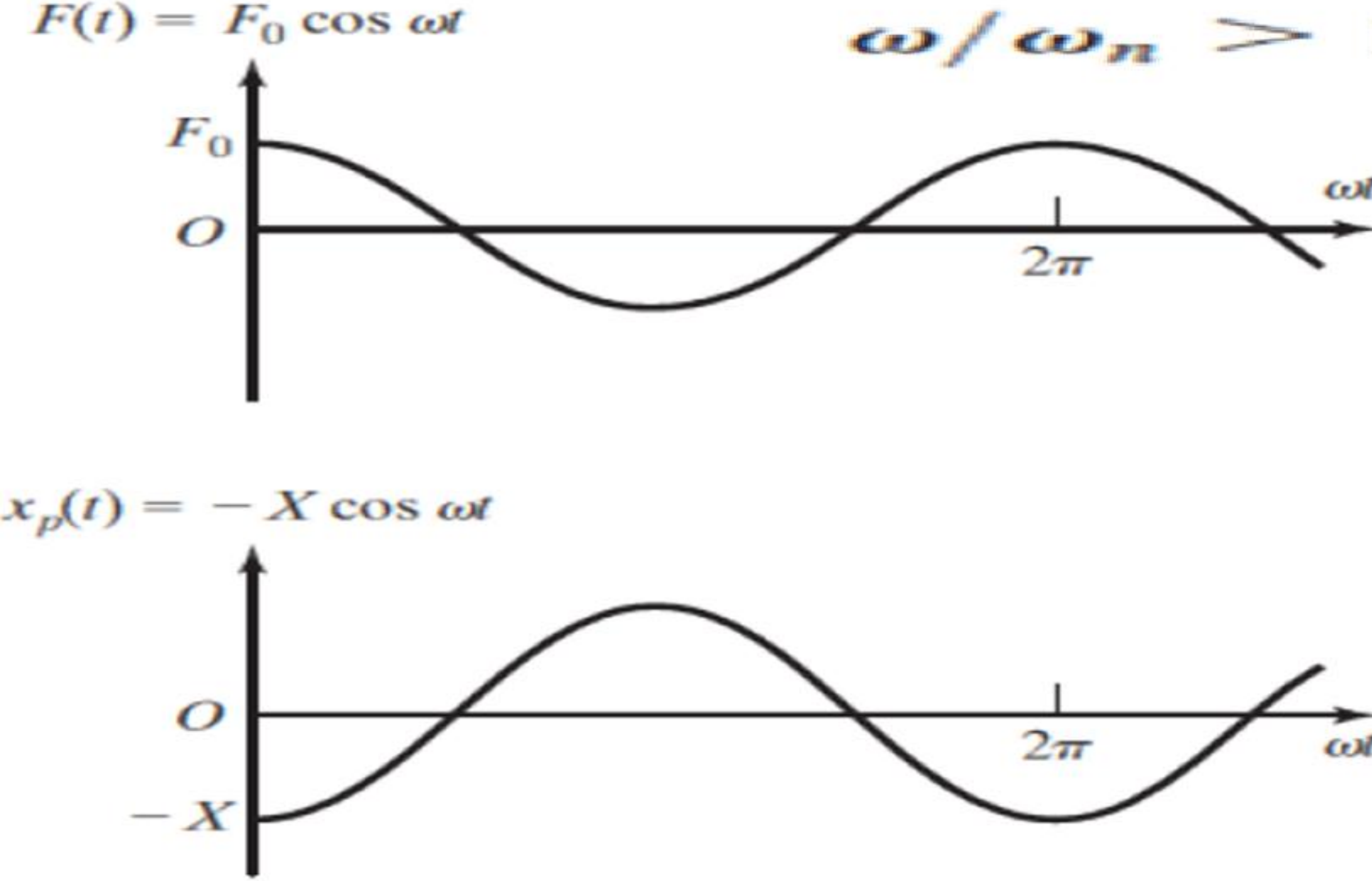
$x_p(t) = X \cos \omega t$



Harmonic response when

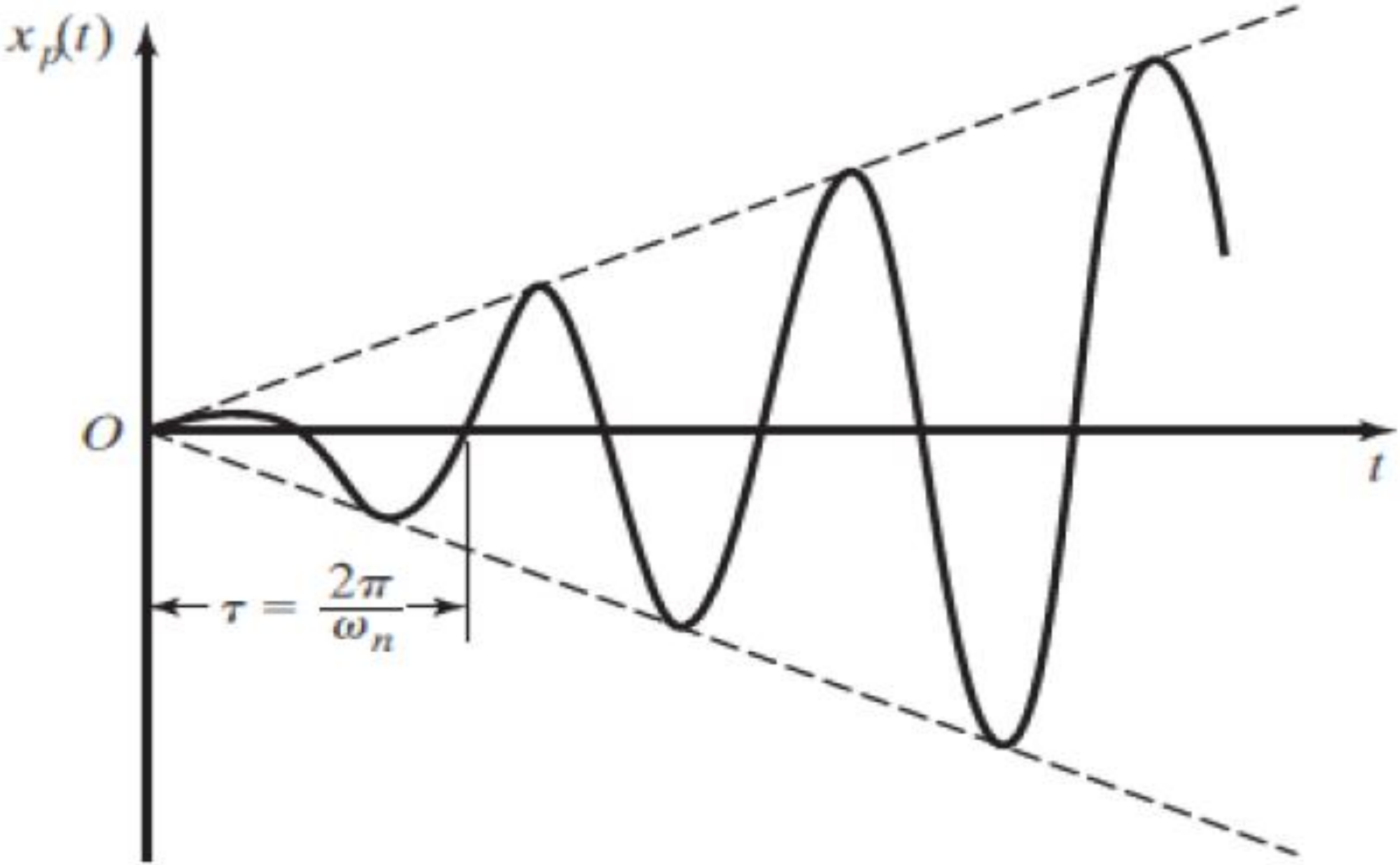
$0 < \omega / \omega_n < 1.$

$\omega / \omega_n > 1.$

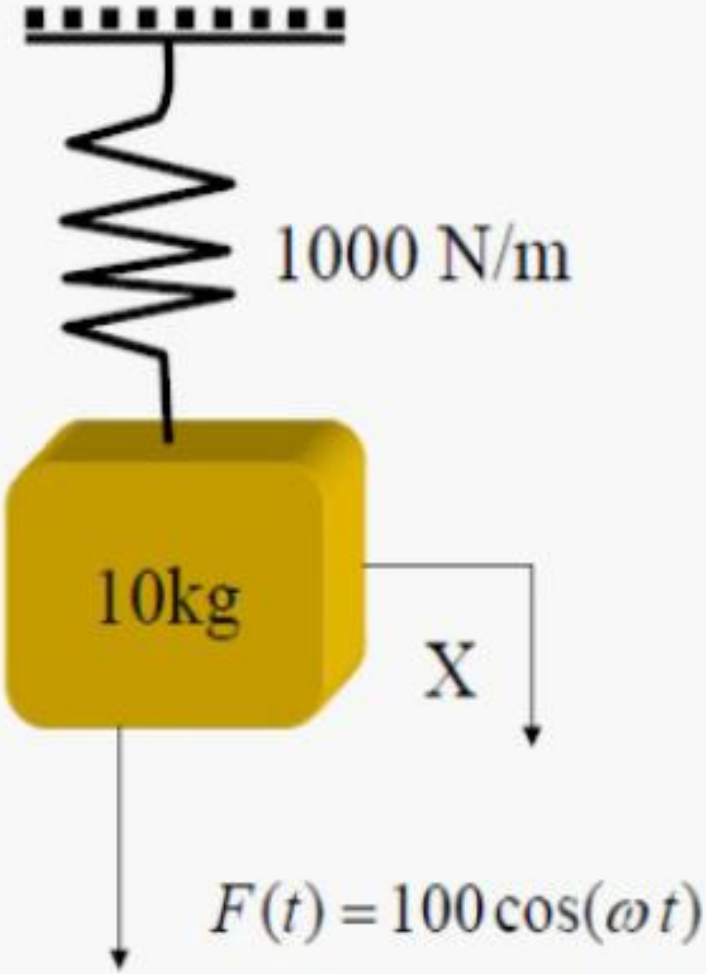


Harmonic response when

$\omega / \omega_n > 1.$



Response when $\omega/\omega_n = 1$.



Example

To Find the Steady State Response

$$x_{ss}(t) = \frac{F_o}{k - m\omega^2}\cos(\omega t)$$

$$x_{ss}(t) = \frac{100}{1000 - 10\omega^2}\cos(\omega t)$$

$$\omega_n = \sqrt{\frac{1000}{10}} = 10 \text{ rad/sec}$$