

Module

2

AC to DC Converters

Lesson

14

Operation and Analysis of Three Phase Half Controlled Converter

Instructional Objectives

On completion the student will be able to

- Draw the circuit diagram and waveforms of different variables associated with a three phase half controlled converter.
- Identify the constructional and operational difference between a three phase fully controlled and half controlled converter.
- Calculate the average and RMS value of the output dc voltage.
- Calculate the displacement factor, distortion factor and power factor of the input ac line current.
- Calculate the Fourier series components of the output voltage and input current waveforms.
- Derive the closed form expression for output dc current and hence identify continuous or discontinuous conduction mode of the converter.

14.1 Introduction

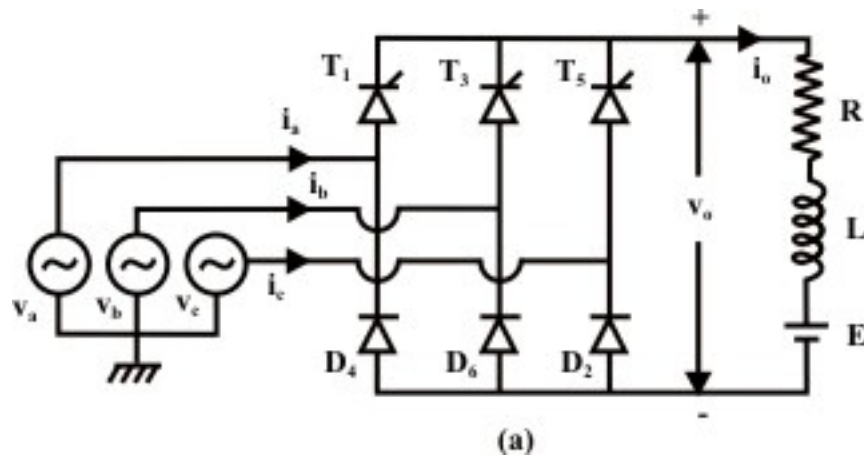
Three phase fully controlled converters are very popular in many industrial applications particularly in situations where power regeneration from the dc side is essential. It can handle reasonably high power and has acceptable input and output harmonic distortion. The configuration also lends itself to easy series and parallel connection for increasing voltage and current rating or improvement in harmonic behavior. However, this versatility of a three phase fully controlled converters are obtained at the cost of increased circuit complexity due to the use of six thyristors and their associated control circuit. This complexity can be considerably reduced in applications where power regeneration is not necessary. In that case three thyristors of the top group or the bottom group of a three phase fully controlled converter can be replaced by three diodes. The resulting converter is called a three phase half controlled converter. Replacing three thyristors by three diodes reduces circuit complexity but at the same time prevents negative voltage appearing at the output at any time. Therefore the converter cannot operate in the inverting mode.

The three phase half controlled converter has several other advantages over a three phase fully controlled converter. For the same firing angle it has lower input side displacement factor compared to a fully controlled converter. It also extends the range of continuous conduction of the converter. It has one serious disadvantage however. The output voltage is periodic over one third of the input cycle rather than one sixth as is the case with fully controlled converters. This implies both input and output harmonics are of lower frequency and require heavier filtering. For this reason half controlled three phase converters are not as popular as their fully controlled counterpart.

Although, from the point of view of construction and circuit complexity the half controlled converter is simpler compared to the fully controlled converter, its analysis is considerably more difficult. In this lesson the operating principle and analysis of a three phase half controlled converter operating in the continuous conduction mode will be presented.

14.2 Operating principle of three phase half controlled converter

Fig. 14.1(a) shows the circuit diagram of three phase half controlled converter supplying an R-L-E load. In the continuous conduction mode only one thyristor from top group and only one diode from the bottom group conduct at a time. However, unlike fully controlled converter here both devices from the same phase leg can conduct at the same time. Hence, there are nine conducting modes as shown in Fig. 14.1(b).



Mode Volt	T ₁ D ₂	D ₂ T ₃	T ₃ D ₄	D ₄ T ₅	T ₅ D ₆	D ₆ T ₁	T ₁ D ₄	T ₃ D ₆	T ₅ D ₂
V _{T1}	0	v _{ab}	v _{ab}	v _{ac}	v _{ac}	0	0	v _{ab}	v _{ac}
V _{D2}	0	0	v _{ac}	v _{ac}	v _{bc}	v _{bc}	v _{ac}	v _{bc}	0
V _{T3}	v _{ba}	0	0	v _{bc}	v _{bc}	v _{ba}	v _{ba}	0	v _{bc}
V _{D4}	v _{ca}	v _{ca}	0	0	v _{ba}	v _{ba}	0	v _{ba}	v _{ca}
V _{T5}	v _{ca}	v _{cb}	v _{cb}	0	0	v _{ca}	v _{ca}	v _{cb}	0
V _{D6}	v _{cb}	v _{cb}	v _{ab}	v _{ab}	0	0	v _{ab}	0	v _{cb}
V ₀	v _{ac}	v _{bc}	v _{ba}	v _{ca}	v _{cb}	v _{ab}	0	0	0

(b)

Fig. 14.1: Circuit diagram and conduction diagram of a three phase half controlled converter
(a) Circuit diagram; (b) Conduction diagram

Now consider the conducting and blocking state of D₂. In the blocking state the voltage across D₂ is either v_{ac} or v_{bc}. Hence, D₂ can block only when these voltages are negative. Taking v_{bc} as the reference phasor (i.e., v_{bc} = √2V_L sin ωt) D₂ will block during 2π/3 ≤ ωt ≤ 2π and will conduct in the interval 0 ≤ ωt ≤ 2π/3. Similarly it can be shown that D₄ and D₆ will conduct during 2π/3 ≤ ωt ≤ 4π/3 and 4π/3 ≤ ωt ≤ 2π respectively.

Next consider conduction of T₁. The firing sequence of the thyristor is T₁ → T₃ → T₅. Therefore before T₁ comes into conduction T₅ conducts and voltage across T₁ is v_{ac} = √2V_L sin(ωt + π/3). If the firing angle of T₁ is α then T₁ starts conduction at ωt = α - π/3 and conducts upto α + π/3. Similarly T₃ and T₅ conducts during α + π/3 ≤ ωt ≤ α + π and α + π ≤ ωt ≤ 2π + α - π/3. From this discussion the following conduction diagrams can be drawn for continuous conduction mode.

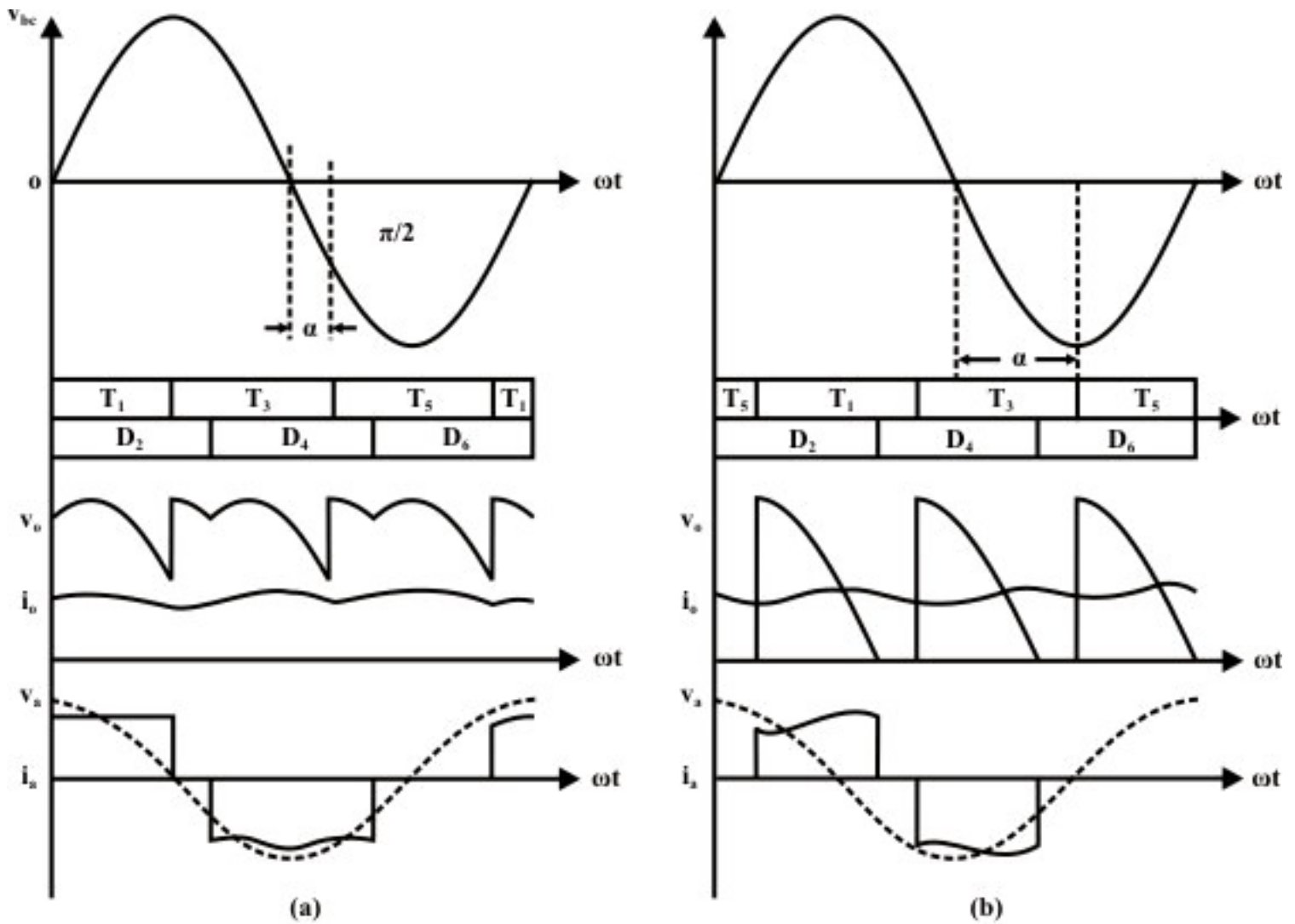


Fig. 14.2: Waveforms of three phase half controlled converter
 (a) $\alpha = \pi/6$; (b) $\alpha = \pi/2$.

Exercise 14.1

Fill in the blanks(s) with appropriate word(s).

- i. A three phase half controlled converter has _____ thyristors and _____ diodes.
- ii. A three phase half controlled converter has _____ conduction modes as compared to _____ of a fully controlled converter.
- iii. A three phase half controlled converter can not operate in the _____ mode.
- iv. Unlike a three phase fully controlled converter the devices in the _____ phase leg of a half controlled converter can conduct at a given time. These conduction modes are called _____ modes.
- v. In a three phase half controlled converter only _____ conduction modes appear at the same time.

- vi. _____ modes appear only when the firing angle of the converter is greater than _____ degrees.
- vii. In a three phase half controlled converter the diodes conduct in a manner similar to a _____ converter whereas the thyristors conduct similar to a _____ converter.
- viii. The input current of a three phase half controlled converter does not have _____ cycle symmetry.

Answer: (i) three, three; (ii) nine, six; (iii) inverter; (iv) same, free wheeling; (v) six; (vi) free wheeling, 60; (vii) uncontrolled, controlled; (viii) quarter.

14.3 Analysis of three phase half controlled converters

Fig. 14.2 (a) and (b) also shows the waveforms of v_0 and i_0 (for $\alpha < \pi/3$ and $\alpha > \pi/3$) both of which are periodic over one third of the input voltage time period. Therefore examining v_0 for the conduction period of any one thyristor (for example T_1) will be sufficient to deduce information regarding output voltage. For example the average value of v_0 can be found as follows.

With T_1 conducting there can be three conduction modes namely, T_1D_6 , T_1D_2 and T_1D_4 .

Now T_1 conducts in the interval $\alpha - \frac{\pi}{3} \leq \omega t \leq \alpha + \frac{\pi}{3}$

D_2 conducts in the interval $0 \leq \omega t \leq 2\frac{\pi}{3}$

D_4 conducts in the interval $2\frac{\pi}{3} \leq \omega t \leq 4\frac{\pi}{3}$

D_6 conducts in the interval $4\frac{\pi}{3} \leq \omega t \leq 2\pi$

\therefore Conduction interval T_1D_6 exists only if $\alpha \leq \frac{\pi}{3}$

Conduction interval T_1D_4 exists only if $\alpha > \frac{\pi}{3}$

So for $\alpha \leq \frac{\pi}{3}$

In the interval $\alpha - \frac{\pi}{3} \leq \omega t \leq \alpha + \frac{\pi}{3}$

$$v_0 = v_{ab} = \sqrt{2}V_L \sin\left(\omega t + \frac{2\pi}{3}\right) \quad (14.1)$$

$$0 \leq \omega t \leq \alpha + \frac{\pi}{3}$$

$$v_0 = v_{ac} = \sqrt{2}V_L \sin\left(\omega t + \frac{\pi}{3}\right) \quad (14.2)$$

$$V_0 = \frac{3\sqrt{2}V_L}{2\pi} \left[\int_{\alpha - \frac{\pi}{3}}^0 \sin\left(\omega t + 2\frac{\pi}{3}\right) d\omega t + \int_0^{\alpha + \frac{\pi}{3}} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t \right]$$

$$= \frac{3\sqrt{2}V_L}{2\pi} \left[\cos\left(\alpha + \frac{\pi}{3}\right) - \cos\frac{2\pi}{3} + \cos\frac{\pi}{3} - \cos\left(\alpha + \frac{2\pi}{3}\right) \right] \quad (14.3)$$

$$\text{or, } V_0 = \frac{3\sqrt{2}}{2\pi} V_L (1 + \cos\alpha) \quad (14.4)$$

$$\text{For } \alpha > \frac{\pi}{3}, \quad \text{In the interval } \alpha - \frac{\pi}{3} \leq \omega t \leq \frac{2\pi}{3}$$

$$v_0 = v_{ac} = \sqrt{2}V_L \sin\left(\omega t + \frac{\pi}{3}\right) \quad (14.5)$$

$$\text{for } \frac{2\pi}{3} \leq \omega t \leq \alpha + \frac{\pi}{3} \\ v_0 = 0 \quad (14.6)$$

$$\therefore V_0 = \frac{3\sqrt{2}V_L}{2\pi} \left[\int_{\alpha - \frac{\pi}{3}}^{2\pi} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t \right] \\ = \frac{3\sqrt{2}}{2\pi} V_L (1 + \cos\alpha)$$

RMS value of v_0 can be found in a similar manner and is left as an exercise.

From the waveforms of Fig. 14.2, v_0 is periodic over one third of the input cycle. Therefore one can write

$$v_0 = V_0 + \sum_{n=1}^{\infty} [V_{An} \cos 3n\omega t + V_{Bn} \sin 3n\omega t] \quad (14.8)$$

$$V_{An} = \frac{3}{\pi} \int_{\alpha - \frac{\pi}{3}}^{\alpha + \frac{\pi}{3}} v_0 \cos 3n\omega t d\omega t \quad (14.9)$$

$$V_{Bn} = \frac{3}{\pi} \int_{\alpha - \frac{\pi}{3}}^{\alpha + \frac{\pi}{3}} v_0 \sin 3n\omega t d\omega t \quad (14.10)$$

$$\text{For } \alpha \leq \frac{\pi}{3} \quad \text{From equations 14.1 and 14.2}$$

$$V_{An} = \frac{3}{\pi} \left[\sqrt{2}V_L \int_{\alpha - \frac{\pi}{3}}^0 \sin\left(\omega t + 2\frac{\pi}{3}\right) \cos 3n\omega t d\omega t + \sqrt{2}V_L \int_0^{\alpha + \frac{\pi}{3}} \sin\left(\omega t + \frac{\pi}{3}\right) \cos 3n\omega t d\omega t \right] \\ = \frac{3\sqrt{2}V_L}{2\pi} \left[\int_{\alpha - \frac{\pi}{3}}^0 \left\{ \sin\left((3n+1)\omega t + 2\frac{\pi}{3}\right) + \sin\left((1-3n)\omega t + 2\frac{\pi}{3}\right) \right\} d\omega t \right. \\ \left. + \int_0^{\alpha + \frac{\pi}{3}} \left\{ \sin\left((3n+1)\omega t + \frac{\pi}{3}\right) - \sin\left((3n-1)\omega t - \frac{\pi}{3}\right) \right\} d\omega t \right]$$

$$= \frac{3\sqrt{2}V_L}{2\pi} \left[\begin{aligned} & \left. \frac{\cos\{(3n+1)\omega t + 2\pi/3\}}{3n+1} \right|_0^{\alpha-\pi/3} + \left. \frac{\cos\{(3n-1)\omega t - 2\pi/3\}}{3n-1} \right|_{\alpha-\pi/3}^0 \\ & + \left. \frac{\cos\{(3n+1)\omega t + \pi/3\}}{3n+1} \right|_{\alpha+\pi/3}^0 + \left. \frac{\cos\{(3n-1)\omega t - \pi/3\}}{3n-1} \right|_0^{\alpha+\pi/3} \end{aligned} \right] \quad (14.11)$$

Therefore

$$\begin{aligned} V_{An} &= \frac{3\sqrt{2}V_L}{2\pi} \left[\frac{1 + \cos[(3n+1)(\alpha - \pi/3) + 2\pi/3] - \cos[(3n+1)(\alpha + \pi/3) + \pi/3]}{3n+1} \right. \\ & \quad \left. + \frac{\cos[(3n-1)(\alpha + \pi/3) - \pi/3] - \cos[(3n-1)(\alpha - \pi/3) - 2\pi/3] - 1}{3n-1} \right] \\ &= \frac{3\sqrt{2}V_L}{2\pi} \left[\frac{1 - 2\sin[(3n+1)\alpha + \pi/2] \sin[\pi/6 - (3n+1)\pi/3]}{3n+1} \right. \\ & \quad \left. - \frac{1 + 2\sin[(3n-1)\alpha - \pi/2] \sin[\pi/6 + (3n-1)\pi/3]}{3n-1} \right] \\ &= \frac{3\sqrt{2}V_L}{2\pi} \left[\frac{1 + 2\sin(6n+1)\pi/6 \cos(3n+1)\alpha}{3n+1} - \frac{1 - 2\sin(6n-1)\pi/6 \cos(3n-1)\alpha}{3n-1} \right] \\ &= \frac{3\sqrt{2}V_L}{2\pi} \left[\frac{1 + (-1)^n \cos(3n+1)\alpha}{3n+1} - \frac{1 + (-1)^n \cos(3n-1)\alpha}{3n-1} \right] \quad (14.12) \end{aligned}$$

Similarly,

$$V_{Bn} = \frac{3\sqrt{2}V_L}{\pi} \left[\int_{\alpha-\pi/3}^0 \sin\left(\omega t + 2\frac{\pi}{3}\right) \sin 3n\omega t d\omega t + \int_0^{\alpha+\pi/3} \sin\left(\omega t + \frac{\pi}{3}\right) \sin 3n\omega t d\omega t \right] \quad (14.13)$$

$$\begin{aligned} \text{or, } V_{Bn} &= \frac{3\sqrt{2}V_L}{2\pi} \left[\int_{\alpha-\pi/3}^0 \left\{ \cos\left[(3n-1)\omega t - 2\frac{\pi}{3}\right] - \cos\left[(3n+1)\omega t + 2\frac{\pi}{3}\right] \right\} d\omega t \right. \\ & \quad \left. + \int_0^{\alpha+\pi/3} \left\{ \cos\left[(3n-1)\omega t - \frac{\pi}{3}\right] - \cos\left[(3n+1)\omega t + \frac{\pi}{3}\right] \right\} d\omega t \right] \\ &= \frac{3\sqrt{2}V_L}{2\pi} \left[\left. \frac{\sin\{(3n-1)\omega t - 2\pi/3\}}{3n-1} \right|_{\alpha-\pi/3}^0 - \left. \frac{\sin\{(3n+1)\omega t + 2\pi/3\}}{3n+1} \right|_{\alpha-\pi/3}^0 \right. \\ & \quad \left. + \left. \frac{\sin\{(3n-1)\omega t - \pi/3\}}{3n-1} \right|_0^{\alpha+\pi/3} - \left. \frac{\sin\{(3n+1)\omega t + \pi/3\}}{3n+1} \right|_0^{\alpha+\pi/3} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{3\sqrt{2}V_L}{2\pi} \left[\frac{\sin[(3n-1)(\alpha + \pi/3) - \pi/3] - \sin[(3n-1)(\alpha - \pi/3) - 2\pi/3]}{3n-1} \right. \\
&\quad \left. - \frac{\sin[(3n+1)(\alpha + \pi/3) + \pi/3] - \sin[(3n+1)(\alpha - \pi/3) + 2\pi/3]}{3n+1} \right] \\
&= \frac{3\sqrt{2}V_L}{\pi} \left[\frac{\cos[(3n-1)\alpha - \pi/2] \sin[\pi/6 + (3n-1)\pi/6]}{3n-1} \right. \\
&\quad \left. - \frac{\cos[(3n+1)\alpha + \pi/2] \sin[(3n+1)\pi/6 - \pi/6]}{3n+1} \right] \\
&= \frac{3\sqrt{2}V_L}{\pi} \left[\frac{\sin(3n+1)\alpha}{3n+1} + \frac{\sin(3n-1)\alpha}{3n-1} \right] \sin \frac{n\pi}{2} \\
\therefore V_{Bn} &= \frac{3\sqrt{2}}{\pi} V_L \sin \frac{n\pi}{2} \left[\frac{\sin(3n+1)\alpha}{3n+1} + \frac{\sin(3n-1)\alpha}{3n-1} \right] \quad (14.14)
\end{aligned}$$

Similar analysis can be done for $\alpha > \frac{\pi}{3}$

To find out the Fourier series of the input ac line current the load may be replaced by a constant current source having the same value as the average load current. This approximation will be valid provided the load current ripple is relatively small. With this assumption the last waveform of Fig. 14.2(b) can be redrawn as follows.

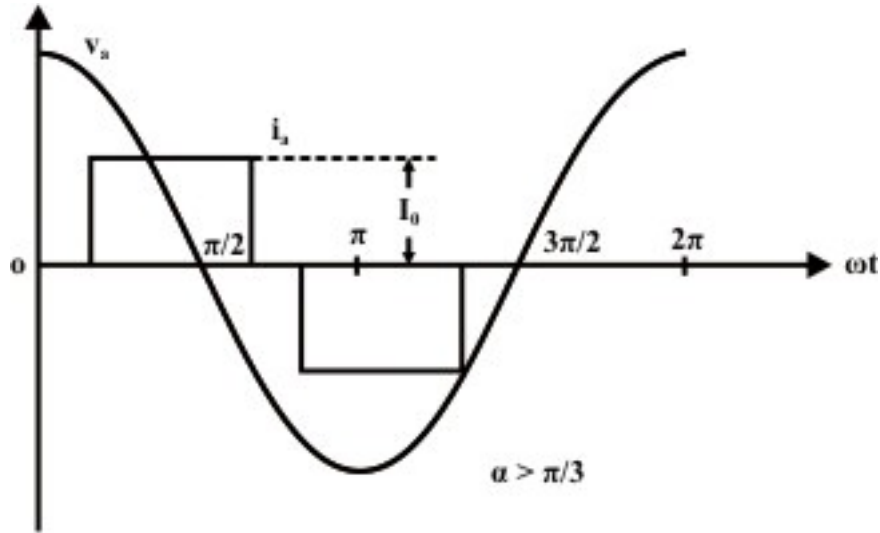


Fig. 14.3: Phase voltage and current waveforms.

$$\begin{aligned}
\therefore \quad \alpha - \frac{\pi}{3} \leq \omega t \leq \frac{2\pi}{3} & \quad i_a = I_0 \\
\alpha + \frac{\pi}{3} \leq \omega t \leq \frac{4\pi}{3} & \quad i_a = -I_0 \\
\text{otherwise} & \quad i_a = 0
\end{aligned}$$

$$i_a = \sum_{n=1}^{\alpha} [I_{an} \cos n\omega t + I_{bn} \sin n\omega t] \quad (14.15)$$

$$\begin{aligned} I_{an} &= \frac{1}{\pi} \int_0^{2\pi} i_a \cos n\omega t \, d\omega t \\ &= \frac{1}{\pi} \left[\int_{\alpha - \frac{\pi}{3}}^{2\frac{\pi}{3}} I_0 \cos n\omega t \, d\omega t - \int_{\alpha + \frac{\pi}{3}}^{4\frac{\pi}{3}} I_0 \cos n\omega t \, d\omega t \right] \\ &= \frac{I_0}{\pi} \left[\frac{\sin n\omega t}{n} \Big|_{\alpha - \frac{\pi}{3}}^{2\frac{\pi}{3}} - \frac{\sin n\omega t}{n} \Big|_{\alpha + \frac{\pi}{3}}^{4\frac{\pi}{3}} \right] \\ &= \frac{I_0}{n\pi} \left[\sin \frac{2n\pi}{3} - \sin n \left(\alpha - \frac{\pi}{3} \right) + \sin n \left(\alpha + \frac{\pi}{3} \right) - \sin \frac{4n\pi}{3} \right] \\ &= \frac{2I_0}{n\pi} \left[\sin \frac{2n\pi}{3} + \cos n\alpha \sin \frac{n\pi}{3} \right] \end{aligned}$$

$$\text{or, } I_{an} = \frac{2I_0}{n\pi} [\cos n\alpha - (-1)^n] \sin \frac{n\pi}{3} \quad (14.16)$$

$$\begin{aligned} I_{bn} &= \frac{1}{\pi} \int_0^{2\pi} i_a \sin n\omega t \, d\omega t \\ &= \frac{1}{\pi} \left[\int_{\alpha - \frac{\pi}{3}}^{2\frac{\pi}{3}} I_0 \sin n\omega t \, d\omega t - \int_{\alpha + \frac{\pi}{3}}^{4\frac{\pi}{3}} I_0 \sin n\omega t \, d\omega t \right] \\ &= \frac{I_0}{n\pi} \left[\cos n\omega t \Big|_{2\frac{\pi}{3}}^{\alpha - \frac{\pi}{3}} + \cos n\omega t \Big|_{\alpha + \frac{\pi}{3}}^{4\frac{\pi}{3}} \right] \\ &= \frac{I_0}{n\pi} \left[\sin \frac{4n\pi}{3} - \cos \frac{2n\pi}{3} + \cos n \left(\alpha - \frac{\pi}{3} \right) - \cos n \left(\alpha + \frac{\pi}{3} \right) \right] \\ &= \frac{2I_0}{n\pi} \sin n\alpha \sin \frac{n\pi}{3} \end{aligned} \quad (14.17)$$

For the fundamental component $n = 1$

$$\begin{aligned} i_{a1} &= \frac{\sqrt{3}I_0}{\pi} [\cos\omega t + \cos\alpha \cos\omega t + \sin\alpha \sin\omega t] \\ &= \frac{\sqrt{3}I_0}{\pi} [\cos\omega t + \cos(\omega t - \alpha)] \\ &= \frac{2\sqrt{3}I_0}{\pi} \cos \frac{\alpha}{2} \cos \left(\omega t - \frac{\alpha}{2} \right) \end{aligned} \quad (14.18)$$

$$\therefore \text{Displacement factor} = \cos \frac{\alpha}{2} \quad (14.19)$$

$$\text{Distortion factor} = \frac{I_{a1}}{I_a} = \frac{\frac{\sqrt{6}}{\pi} I_0 \cos \frac{\alpha}{2}}{I_0 \sqrt{\frac{\pi - \alpha}{\pi}}} = \sqrt{\frac{6}{\pi(\pi - \alpha)}} \cos \frac{\alpha}{2} \quad (14.20)$$

∴ Power factor = Distortion factor × Displacement factor

$$\sqrt{\frac{6}{\pi(\pi - \alpha)}} \cos^2 \frac{\alpha}{2} = \frac{\sqrt{3}}{\sqrt{2(\pi - \alpha)\pi}} (1 + \cos\alpha) \quad (14.21)$$

A closed form expression for i_0 can be found as follows

In the interval $0 < \omega t \leq \alpha + \frac{\pi}{3}$ $v_0 = v_{ac}$

$$\therefore L \frac{di_0}{dt} + Ri_0 + E = v_{ac} = \sqrt{2}V_L \sin\left(\omega t + \frac{\pi}{3}\right) \quad (14.22)$$

$$\therefore i_0 = Ie^{-\frac{\omega t}{\tan\phi}} + \frac{\sqrt{2}V_L}{Z} \left[\sin\left(\omega t + \frac{\pi}{3} - \phi\right) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.23)$$

$$\text{Where } \tan\phi = \frac{\omega L}{R}, \quad Z = \sqrt{R^2 + \omega^2 L^2} \quad \text{and } E = \sqrt{2}V_L \sin\theta \quad (14.24)$$

At $\omega t = \alpha + \frac{\pi}{3}$

$$i_0 = I_1 = Ie^{-\frac{(\alpha + \pi/3)}{\tan\phi}} + \frac{\sqrt{2}V_L}{Z} \left[\sin\left(\alpha - \phi + \frac{2\pi}{3}\right) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.25)$$

In the interval $\alpha + \frac{\pi}{3} \leq \omega t \leq \frac{2\pi}{3}$ $v_0 = v_{bc}$

$$\therefore L \frac{di_0}{dt} + Ri_0 + E = v_{bc} = \sqrt{2}V_L \sin\omega t \quad (14.26)$$

$$\therefore i_0 = I_2 e^{-\frac{(\omega t - \alpha - \pi/3)}{\tan\phi}} + \frac{\sqrt{2}V_L}{Z} \left[\sin(\omega t - \phi) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.27)$$

At $\omega t = \alpha + \frac{\pi}{3}$

$$i_0 = I_2 + \frac{\sqrt{2}V_L}{Z} \left[\sin\left(\alpha + \frac{\pi}{3} - \phi\right) - \frac{\sin\theta}{\cos\phi} \right] = I_1 \quad (14.28)$$

$$\therefore I_2 = Ie^{-\frac{(\alpha + \pi/3)}{\tan\phi}} - \frac{\sqrt{2}V_L}{Z} \sin(\alpha - \phi) \quad (14.29)$$

$$\therefore i_0 = Ie^{-\frac{(\omega t)}{\tan\phi}} + \frac{\sqrt{2}V_L}{Z} \left[\sin(\phi - \alpha) e^{-\frac{(\omega t - \alpha - \pi/3)}{\tan\phi}} + \sin(\omega t - \phi) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.30)$$

$$i_0 \Big|_{\omega t = \frac{2\pi}{3}} = Ie^{-\frac{2\pi}{3\tan\phi}} + \frac{\sqrt{2}V_L}{Z} \left[\sin(\phi - \alpha) e^{\frac{(\alpha - \pi/3)}{\tan\phi}} - \sin\left(\phi - \frac{2\pi}{3}\right) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.31)$$

$$i_0 \Big|_{\omega t = \frac{2\pi}{3}} = i_0 \Big|_{\omega t = 0} = I + \frac{\sqrt{2}V_L}{Z} \left[\sin\left(\frac{\pi}{3} - \phi\right) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.32)$$

$$\therefore I \left(1 - e^{-\frac{2\pi}{3\tan\phi}} \right) = \frac{\sqrt{2}V_L}{Z} \left[\sin(\phi - \alpha) e^{\frac{(\alpha - \pi/3)}{\tan\phi}} + \sin\phi \right]$$

$$\therefore \text{for } 0 < \omega t \leq \alpha + \frac{\pi}{3}$$

$$i_0 = \frac{\sqrt{2}V_L}{Z} \left[\sin(\phi - \alpha) \frac{e^{-\frac{(\omega t - \alpha - \pi/3)}{\tan\phi}}}{1 - e^{-\frac{2\pi}{3\tan\phi}}} + \frac{\sin\phi e^{-\frac{\omega t}{\tan\phi}}}{1 - e^{-\frac{2\pi}{3\tan\phi}}} + \sin\left(\omega t + \frac{\pi}{3} - \phi\right) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.33)$$

$$\text{for } \alpha + \frac{\pi}{3} \leq \omega t \leq \frac{2\pi}{3}$$

$$i_0 = \frac{\sqrt{2}V_L}{Z} \left[\sin(\phi - \alpha) \left\{ e^{-\frac{(\omega t - \alpha - \pi/3)}{\tan\phi}} + \frac{e^{\frac{\alpha - \pi/3 - \omega t}{\tan\phi}}}{1 - e^{-\frac{2\pi}{3\tan\phi}}} \right\} + \frac{\sin\phi e^{-\frac{\omega t}{\tan\phi}}}{1 - e^{-\frac{2\pi}{3\tan\phi}}} + \sin(\omega t - \phi) - \frac{\sin\theta}{\cos\phi} \right] \quad (14.34)$$

Exercise 14.2

1. Fill in the blank(s) with the appropriate word(s).

- In a three phase half controlled converter each thyristor and diode conduct for _____ degrees.
- The output voltage waveform of a three phase half controlled converter is periodic over _____ of the input voltage cycle.
- The output voltage waveform of a three phase half controlled converter operating with $\alpha > \pi/3$ and $\alpha \leq \pi/3$ are _____ and have _____ formula for the average voltage.
- The output voltage and current of a three phase half controlled converter contain _____ harmonics of the input ac frequency.
- The ac input current of a half controlled three phase converter can be zero for larger than _____ of the input ac cycle provided the value of α is _____ than 60° .
- The input ac current of a three phase half controlled converter contain _____ harmonics but no _____ harmonics.
- For the same output load current and firing angle the three phase half controlled converter has better _____ factor but poorer _____ factor compared to a fully controlled converter.

Answer: (i) 120° ; (ii) one third; (iii) different, same; (iv) triplen; (v) one third, greater; (vi) even, triplen; (vii) displacement, distortion.

2. A 200V, 1450 RPM, 100A separately excited dc machine has an armature resistance of 0.04Ω . The machine is driven by a three phase half controlled converter operating from a three phase 220V, 50Hz supply. The motor operates at the rated speed and rated load torque. Assuming continuous conduction find out (i) the firing angle of the converter; (ii) RMS fundamental component of the input current, (iii) Input current displacement factor and distortion factors.

Answer:

(i) Under rated operating condition the motor must be supplied with rated voltage.

$$\text{Therefore } V_o = \frac{3\sqrt{2}}{2\pi} V_L (1 + \cos\alpha) = 200V$$

$$\text{Where } V_L = 230V$$

$$\therefore \alpha \approx 70^\circ$$

(ii) $I_o = 100A$

From equation (14.18)

$$I_{i1} = \frac{\sqrt{6}}{\pi} I_o \cos \frac{\alpha}{2} = 63.87 \text{ amps}$$

(iii) From equation (14.19)

$$\text{Input displacement factor} = \cos \frac{\alpha}{2} = 0.819$$

From equation (14.20)

$$\text{Input distortion factor} = \sqrt{\frac{6}{\pi(\pi - \alpha)}} \cos \frac{\alpha}{2} = 0.712$$

References

1. "Power Electronics" P.C. Sen, Tata McGrawhill publishing company limited, 1995.
2. "Power Electronics, Converters, Applications and Design"; Mohan, Undeland, Robins; John Willey and Sons Inc, Third Edition, 2003.

Lesson Summary

- Three phase half controlled converters are obtained by replacing three thyristors of either the top group or the bottom group of fully controlled converters by three diodes.
- Three phase half controlled converters can not operate in the inverting mode.
- Three phase half controlled converters have nine operating modes as compared to six of a fully controlled converter.
- The three free wheeling modes of a half controlled converters appears only when the firing angle is larger than 60° .
- The output voltage and current waveforms of a three phase half controlled converter consist of a dc component and triplen harmonics of the input voltage frequency.
- For the same input ac voltage and firing angle a half controlled converter has higher output average dc voltage compared to a fully controlled converter.
- The input ac line current of a three phase half controlled converter contains harmonics of all (odd and even) order except triplen harmonics.
- For the same average dc load current and firing angle the half controlled converter has better input current displacement factor but poorer distortion factor compared to a fully controlled converter.
- The triggering circuit of a three phase half controlled converter is similar to that of a fully controlled converter. However, only three are required.

Practice Problems and Answers

1. If a free wheeling diode is connected across the output terminals of a three phase fully controlled converter will the performance of converter will be similar to a half controlled converter? Justify your answer.
2. A 220V, 1500 rpm, 50A, separately excited dc motor with armature resistance of 0.5Ω if fed from a 3 phase half controlled rectifier. The available ac source is 440V, 50Hz. A star delta connected transformer is used to feed the armature so that the motor terminal voltage equals rated voltage when converter firing angle is zero.
 - (i) Calculate the transformer turns ratio
 - (ii) Firing angle when (a) motor is running at 1200 rpm and rated torque; (b) 1500 rpm and half the rated torque.
3. A battery with a nominal voltage of 200V and internal resistance of $10m\Omega$ has to be charged at a constant current of 20 amps from a 3 phase 220V 50 Hz power supply. Which of the following converters will give better performance with respect to input current displacement factor, distortion factor and power factor?
 - (i) 3 phase fully controlled converter; (ii) 3 phase half controlled converter.

Answers to Practice Problems

1) Connecting a diode at the output of a three phase fully controlled converter will not make it performs as a half controlled converter. For example

- i) When $\alpha \leq \pi/3$ the free wheeling diode will not come into conduction and therefore, the converter will continue to perform like a fully controlled converter which is very different from that of a half controlled converter for this range of α .
- ii) For $\alpha > \pi/3$ the output voltage will be clamped to zero for certain part of the input cycle. However, the output voltage will still have “six pulse” characteristics unlike a half controlled converter. Similarly the input current waveform will retain its quarter cycle symmetry which is not the case with a half controlled converter.

2) For a half controlled converter

$$i) \quad V_0 = \frac{3\sqrt{2}}{2\pi} V_L (1 + \cos\alpha)$$

$$\text{at } \alpha = 0, \quad V_0 = 220 \text{ V}, \quad \therefore V_L = 163 \text{ V},$$

$$\text{supply voltage} = 440 \text{ V}, \quad \therefore \text{Primary phase voltage} = 254 \text{ V}$$

$$\therefore \text{Turns ratio} = 1 : 0.64.$$

$$ii) \quad (a) \quad E_b|_{1500} = 220 - 0.5 \times 50 = 195 \text{ V}$$

$$\therefore E_b|_{1200} = 195 \times \frac{12}{15} = 156 \text{ V}$$

$$\text{Torque is rated, } \therefore I_a = 50 \text{ A, } V_{1200} = 156 + 0.5 \times 50 = 181 \text{ volts}$$

$$\therefore 181 = \frac{3\sqrt{2}}{2\pi} \times 163(1 + \cos\alpha) \quad \therefore \alpha = 49.87^\circ$$

$$(b) \quad V_{1500} \text{ at half rated torque} = 195 + 0.5 \times 25 = 207.5 \text{ V}$$

$$207.5 = \frac{3\sqrt{2}}{2\pi} \times 163(1 + \cos\alpha) \quad \therefore \alpha = 27.7^\circ$$

3) The output voltage of the converter should be

$$V_0 = 200 + 20 \times 10 \times 10^{-3} = 200.2 \text{ V}$$

(i) with a fully controlled converter

$$200.2 = \frac{3\sqrt{2}}{\pi} \times 220 \cos\alpha \quad \therefore \alpha = 47.64^\circ$$

$$\therefore \text{Displacement factor} = \cos \alpha = 0.674$$

$$\text{Distortion factor} = \frac{3}{\pi} = 0.955$$

$$\therefore \text{Power factor} = \text{Displacement factor} \times \text{Distortion factor} = 0.6436$$

(ii) with a half controlled converter

$$200.2 = \frac{3\sqrt{2}}{2\pi} \times 220 (1 + \cos\alpha) \quad \therefore \alpha = 69.65^\circ$$

$$\therefore \text{Displacement factor} = \cos \frac{\alpha}{2} = 0.82$$

$$\text{Distortion factor} = \sqrt{\frac{6}{\pi(\pi-\alpha)}} \cos \frac{\alpha}{2} = 0.8166$$

$$\therefore \text{Power factor} = 0.6695$$

\therefore Displacement factor and power factor of a half controlled converter are better compared to a fully controlled converter while the distortion factor is poorer.