AC to DC Converters

Module 2

AC to DC Converters

Lesson 12

Single Phase Uncontrolled Rectifier

Operation and Analysis of three phase uncontrolled rectifier.

Instructional Objectives

On completion the student will be able to

- Draw the conduction table and waveforms of a three phase half wave uncontrolled converter supplying resistive and resistive inductive loads.
- Calculate the average and RMS values of the input / output current and voltage waveforms of a three phase uncontrolled half wave converter.
- Analyze the operation of a three phase full wave uncontrolled converter to find out the input / output current and voltage waveforms along with their RMS and Average values.
- Find out the harmonic components in the input / output voltage and current waveforms of a three phase uncontrolled full wave converter.
- Analyze the operation of a three phase full wave uncontrolled converter supplying a Capacitive – Resistive load.

12.1 Introduction

Single phase rectifiers, as already discussed, are extensively used in low power applications particularly for power supplies to electronic circuits. They are also found useful for supplying small dc loads rarely exceeding 5 KW. Above this power level three phase ac – dc power supplies are usually employed. Single phase ac – dc converters have several disadvantages such as

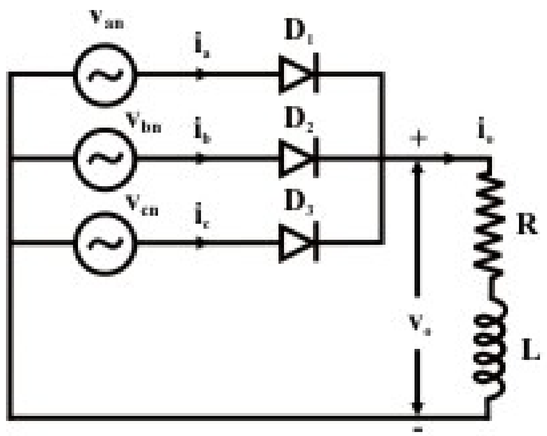
- Large output voltage and current form factor.
- Large low frequency harmonic ripple current causing harmonic power loss and reduced efficiency.
- Very large filter capacitor for obtaining smooth output dc voltage.
- Low frequency harmonic current is injected in the input ac line which is difficult to filter. The situation becomes worse with capacitive loads.

Many of these disadvantages are mitigated to a large extent by using three phase ac – dc converters. In a way it is also natural that bulk loads are supplied by three phase converters since bulk electrical power is always transmitted and distributed in three phases and high power should load three phases symmetrically. Polyphase rectifiers produce less ripple output voltage and current compared to single phase rectifiers. The efficiency of polyphase rectifier is also higher while the associated equipments are smaller.

A three phase supply gives the choice of a number of circuits. These can be placed in one of two groups according to whether three or six diodes are used. These topologies will be analyzed in detail in this section.

12.2 Operating principle of three phase half wave uncontrolled rectifier

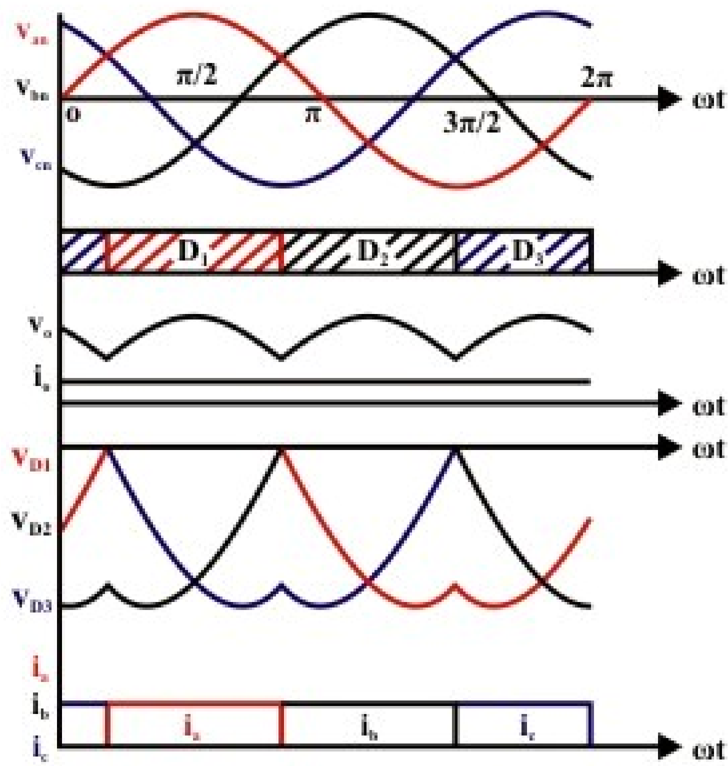
The half wave uncontrolled converter is the simplest of all three phase rectifier topologies. Although not much used in practice it does provide useful insight into the operation of three phase converters. Fig. 12.1 shows the circuit diagram, conduction table and wave forms of a three phase half wave uncontrolled converter supplying a resistive inductive load.



(a)

Voltage / Mode	V_{D1}	V_{D2}	V_{D3}	V_o	
D_1	0	v_{ba}	v_{ca}	v_{an}	
D_2	v_{ab}	0	v_{cb}	v_{bn}	
D_3	v_{ac}	v_{bc}	0	v_{cn}	

(b)



(c)

Fig. 12.1: Operation of the three phase half wave uncontrolled rectifier (a) circuit diagram; (b) conduction table; (c) wave forms.

For simplicity the load current (i_o) has been assumed to be ripple free. As shown in Fig. 12.1 (a), in a three phase half wave uncontrolled converter the anode of a diode is connected to each phase voltage source. The cathodes of all three diodes are connected together to form the positive load terminal. The negative terminal of the load is connected to the supply neutral.

Fig. 12.1 (b) shows the conduction table of the converter. It should be noted that for the type of load chosen the converter always operates in the continuous conduction mode. The conduction diagram for the diodes (as shown in Fig. 12.1 (c) second waveform) can be drawn easily from the conduction diagram. Since the diodes can block only negative voltage it follows from the conduction table that a phase diode conducts only when that phase voltage is maximum of the three. (In signal electronics the circuit of Fig. 12.1 (a) is also known as the "maximum value" circuit). Once the conduction diagram is drawn other waveforms of Fig. 12.1 (c) are easily obtained from the supply voltage waveforms in conjunction with the conduction table.

The phase current waveforms of Fig. 12.1 (c) deserve special mention. All of them have a dc component which flows through the ac source. This may cause "dc saturation" in the ac side transformer. This is one reason for which the converter configuration is not preferred very much in practice.

From the waveforms of Fig. 12.1 (c)

$$V_{OAV} = \frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} \sqrt{2}V_i \sin\omega t \, d(\omega t)$$

$$= \frac{3\sqrt{6}}{2\pi} V_i \qquad (12.1)$$

$$V_{ORMS} = \left[\frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} 2V_i^2 \sin^2\omega t \, d(\omega t)\right]^{\frac{1}{2}}$$

$$= \left[1 + \frac{3\sqrt{3}}{4\pi}\right]^{\frac{1}{2}} V_i \qquad (12.2)$$

$$\therefore \text{ The output voltage form factor} = \frac{V_{ORMS}}{V_{OAV}} = 1.01 \qquad (12.3)$$

$$I_O\big|_{av} = \frac{V_{OAV}}{R},$$

$$I_{i\,RMS} = I_{a\,RMS} = I_{b\,RMS} = I_{c\,RMS} = \frac{I_O}{\sqrt{3}} \qquad (12.4)$$

$$\therefore \text{ Input power factor} = \frac{V_O\big|_{av} I_O\big|_{AV}}{3V_i\, I_{i\,RMS}} = \frac{\frac{3\sqrt{6}}{2\pi}\cancel{V_i}\cancel{I_O}}{3\cancel{V_i}\frac{\cancel{I_O}}{\sqrt{3}}} = \frac{3}{\sqrt{2}\pi} \qquad (12.5)$$

The harmonics present in v_o and i_i can be found by Fourier series analysis of the corresponding waveforms of Fig. 12.1 (c) and is left as an exercise.

Exercise 12.1

Fill in the blank(s) with the appropriate word(s).

i) Three phase half wave uncontrolled rectifier uses ________ diodes.

ii) Three phase half wave uncontrolled rectifier requires ________ phase ______ wire power supply.

iii) In a three phase half wave uncontrolled rectifier each diode conduct for _________ radians.

iv) The minimum frequency of the output voltage ripple in a three phase half wave uncontrolled rectifier is _________ times the input voltage frequency.

v) The input line current of a three phase half wave uncontrolled rectifier contain ________ component.

Answers: (i) three; (ii) three, four; (iii) 2π/3; (iv) three; (v) dc.

2. Assuming ripple free output current, find out the, displacement factor, distortion factor and power factor of a three phase half wave rectifier supplying an R – L load.

With reference to Fig 12.1 the expression for phase current i_a can be written as

$$i_a = I_d \qquad \frac{\pi}{6} \leq \omega t \leq \frac{5\pi}{6}$$

$$i_a = 0 \qquad \text{otherwise.}$$

Fundamental component of i_a can be written as

$$i_{a1} = \sqrt{2}\, I_{a1} \sin(\omega t + \phi)$$

where $\sqrt{2}\, I_{a1} = \sqrt{A_1^2 + B_1^2}$ and $\phi = \tan^{-1}\frac{A_1}{B_1}$

$$A_1 = \frac{1}{\pi}\int_0^{2\pi} i_a \cos\omega t \, d\omega t$$

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} i_a \sin\omega t \, d\omega t$$

$$\therefore \quad A_1 = \frac{1}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} I_d \cos\omega t \, d\omega t = 0$$

$$B_1 = \frac{1}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} I_d \sin\omega t \, d\omega t = \frac{\sqrt{3}}{\pi} I_d$$

$$\therefore \sqrt{2} I_{a1} = B_1 = \frac{\sqrt{3}}{\pi} I_d \qquad \therefore I_{a1} = \sqrt{\frac{3}{2}} \frac{I_d}{\pi}$$

$\phi = 0 \quad \therefore$ Displacement factor $= \cos\phi = 1$.

R.M.S value of $i_a = I_a = \frac{I_d}{\sqrt{3}}$

$\therefore$ Distortion factor $= \frac{I_{a1}}{I_a} = \frac{3}{\sqrt{2}\pi}$

Power Factor = Disp. Factor × Dist. Factor $= \frac{3}{\sqrt{2}\pi}$

12.3 Three phase full wave uncontrolled converter

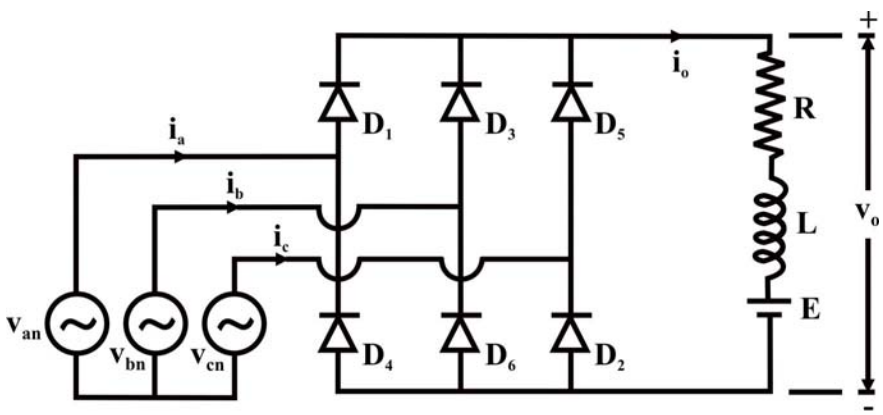
As has been explained earlier three phase half wave converter suffers from several disadvantages. Chief among them are dc component in the input ac current, requirement of neutral connection and comparatively lower output voltage. In addition the input and output waveforms contain lower order harmonics which require heavy filtering.

Most of these disadvantages can be mitigated by using a three phase full wave bridge rectifier. This is probably the most extensively used rectifier topology from low (>5 KW) to moderately high power (> 100 KW) applications. In this section the operation of a three phase full wave uncontrolled bridge rectifier with two different types of loads namely the R – L – E type load and the capacitive load will be described.

12.3.1 Operation of a 3 phase full wave uncontrolled bridge rectifier supplying an R – L – E load

This type of load may represent a dc motor or a battery. Usually for driving these loads a variable output voltage is required. This requirement has to be met by using a variable ac source (e.g a 3 phase variable) since the average output voltage of an uncontrolled rectifier is constant for a given ac voltage.

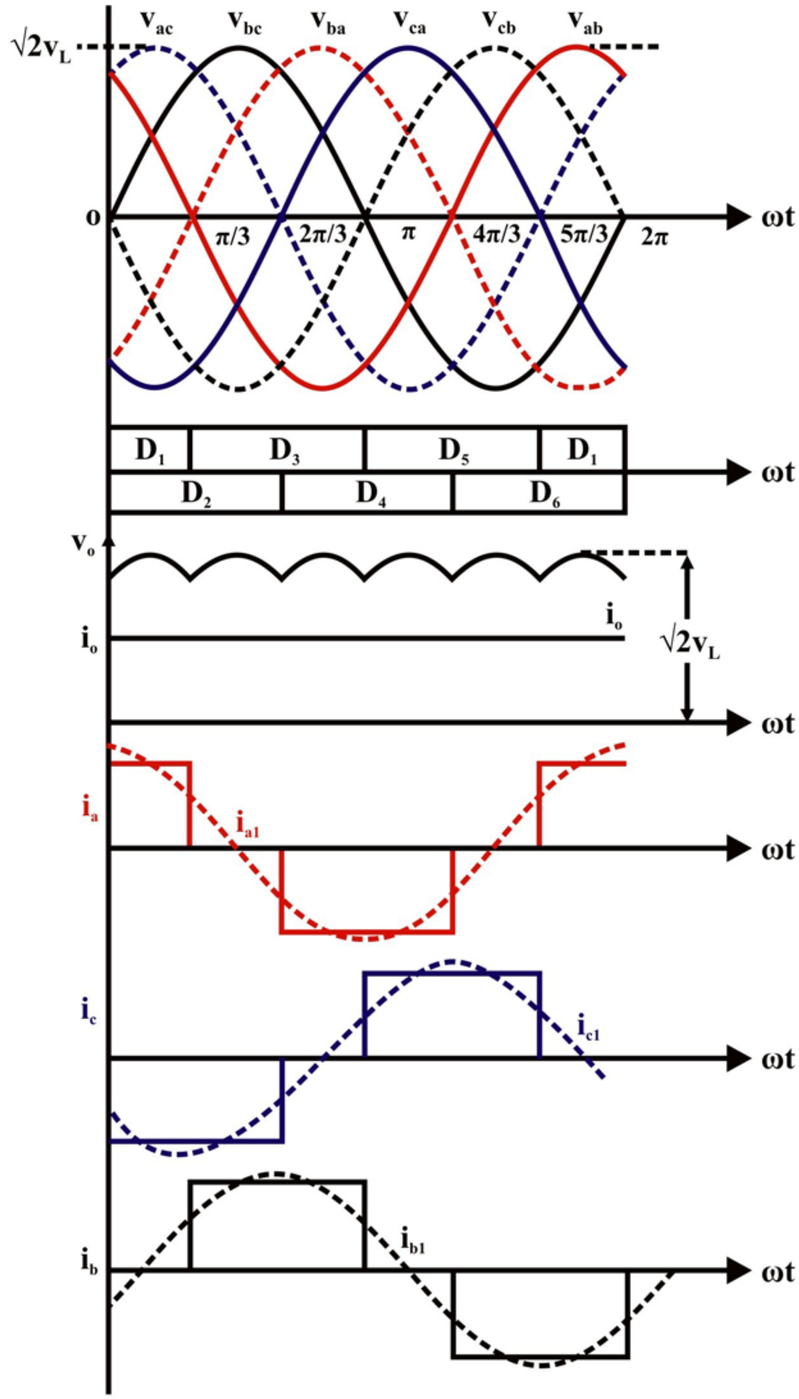
It will also be assumed in the following analysis that the load side inductance is large enough to keep the load current continuous. The relevant condition for continuous conduction will be derived but analysis of discontinuous conduction mode will not be attempted. Compared to single phase converters the cases of discontinuous conduction in 3 phase bridge converter are negligible.



(a)

Device / Mode	v_{D1}	v_{D2}	v_{D3}	v_{D4}	v_{D5}	v_{D6}	v_o
$D_1.D_2$	0	0	v_{ba}	v_{ca}	v_{ca}	v_{cb}	v_{ac}
$D_2.D_3$	v_{ab}	0	0	v_{ca}	v_{cb}	v_{cb}	v_{bc}
$D_3.D_4$	v_{ab}	v_{ac}	0	0	v_{cb}	v_{ab}	v_{ba}
$D_4.D_5$	v_{ac}	v_{ac}	v_{bc}	0	0	v_{ab}	v_{ca}
$D_5.D_6$	v_{ac}	v_{bc}	v_{bc}	v_{ba}	0	0	v_{cb}
$D_6.D_1$	0	v_{bc}	v_{ba}	v_{ba}	v_{ca}	0	v_{ab}

(b)



(c)

Fig. 12.2: Operation of the 3-phase full wave uncontrolled bridge rectifier
(a) circuit diagram
(b) conduction table
(c) waveforms

Since the load current is assumed to be continuous at least one diode from the top group (D_1, D_3 and D_5) and one diode from the bottom group (D_2, D_4 and D_6) must conduct at all time. It can be easily verified that only one diode from each group (either top or bottom) conducts at a time and two diodes from the same phase leg never conducts simultaneously. Thus the converter has six different diode conduction modes. These are D_1D_2, D_2D_3, D_3D_4, D_4D_5, D_5D_6 and D_6D_1. Each conduction mode lasts for $\pi/3$ rad and each diode conducts for 120º.

Fig. 12.2 (b) shows voltages across different diodes and the output voltage in each of these conduction modes. The time interval during which a particular conduction mode will be effective can be ascertained from this table. For example the D_1D_2 conduction mode will occur when the voltage across all other diodes (i.e. v_{ba}, v_{ca} and v_{cb}) are negative. This implies that D_1D_2 conducts in the interval $0 \le \omega t \le \pi/3$ as shown in Fig. 12.2 (c). The diodes have been numbered such that the conduction sequence is $D_1 \rightarrow D_2 \rightarrow D_3 \rightarrow D_4 \rightarrow D_5 \rightarrow D_6 \rightarrow D_1$---. When a diode stops conduction its current is commutated to another diode in the same group (top or bottom). This way the sequence of conduction modes become, $D_1D_2 \rightarrow D_2D_3 \rightarrow D_3D_4 \rightarrow D_4D_5 \rightarrow D_5D_6 \rightarrow D_6D_1 \rightarrow D_1D_2$ ---. The conduction diagram in Fig. 12.2 (c) is constructed accordingly.

The output dc voltage can be constructed from this conduction diagram using appropriate line voltage segments as specified in the conduction table.

The input ac line currents can be constructed from the conduction diagram and the output current. For example

$$
\begin{aligned}
i_a &= i_o && \text{for } 0 \le \omega t \le \pi/3 \text{ and } 5\pi/3 \le \omega t \le 2\pi \\
i_a &= -i_o && \text{for } 2\pi/3 \le \omega t \le 4\pi/3 \\
i_a &= 0 && \text{otherwise.}
\end{aligned} \tag{12.6}
$$

The line current wave forms and their fundamental components are shown in Fig. 12.2 (c).

It is clear from Fig 12.2 (c) that the dc voltage output is periodic over one sixth of the input ac cycle.

For $\pi/3 \le \omega t \le 2\pi/3$

$$v_o = \sqrt{2} V_L \sin \omega t \tag{12.7}$$

$$V_{OAV} = \frac{3}{\pi}\int_{\pi/3}^{2\pi/3} \sqrt{2} V_L \sin \omega t \, d\omega t = \frac{3\sqrt{2}}{\pi} V_L \tag{12.8}$$

$$
\begin{aligned}
V_{ORMS} &= \sqrt{\frac{3}{\pi}\int_{\pi/3}^{2\pi/3} 2V_L^2 \sin^2\omega t \, d\omega t} \\
&= \sqrt{\left(1+\frac{3\sqrt{3}}{2\pi}\right)} V_L
\end{aligned} \tag{12.9}
$$

$$I_{i\,RMS} = \sqrt{\frac{2}{3}}\, I_{OAV}; \quad I_{OAV} = \frac{V_{OAV} - E}{R} \tag{12.10}$$

I_{i1} RMS can be found as follows

$$\sqrt{3}\, V_L\, I_{i1} = V_{OAV}\, I_{OAV} \tag{12.11}$$

Since input displacement factor is unity

$$\therefore \quad I_{i1} = \frac{V_{OAV}}{\sqrt{3}V_L} I_{OAV} = \frac{\sqrt{6}}{\pi} I_{OAV} \tag{12.12}$$

$$\therefore \text{ Power factor} = \text{distortion factor} = \frac{I_{i1}}{I_{i\,RMS}} = \frac{3}{\pi} \tag{12.13}$$

A closed form expression for i_o can be found as follows

for $\quad \pi/3 \leq \omega t \leq 2\pi/3$

$$L\frac{di_o}{dt} + Ri_o + E = v_o = \sqrt{2}V_L \sin\omega t \tag{12.14}$$

The general solution is given by

$$i_o = I_1 e^{-\frac{\omega t - \pi/3}{\tan\varphi}} + \frac{\sqrt{2}V_L}{Z}\left[\sin(\omega t - \varphi) - \frac{\sin\theta}{\cos\varphi}\right] \tag{12.15}$$

Where $\tan\varphi = \frac{\omega L}{R}; \quad \sin\theta = \frac{E}{\sqrt{2}V_L}; \quad Z = \sqrt{R^2 + \omega^2 L^2}$

Now since the current waveform is periodic over one sixth of the input ac cycle

$$i_o\left(\omega t = \frac{\pi}{3}\right) = i_o\left(\omega t = \frac{2\pi}{3}\right) \tag{12.16}$$

$$\therefore \quad I_1 + \frac{\sqrt{2}V_L}{Z}\left[\sin\left(\frac{\pi}{3} - \varphi\right) - \frac{\sin\theta}{\cos\varphi}\right] = I_1 e^{-\frac{\pi}{3\tan\varphi}} + \frac{\sqrt{2}V_L}{Z}\left[\sin\left(\frac{2\pi}{3} - \varphi\right) - \frac{\sin\theta}{\cos\varphi}\right] \tag{12.17}$$

$$\therefore \quad I_1 = \frac{\sqrt{2}V_L}{Z} \frac{\sin\phi}{1 - e^{-\frac{\pi}{3\tan\varphi}}} \tag{12.18}$$

$$\therefore \quad i_o = \frac{\sqrt{2}V_L}{Z}\left[\frac{\sin\varphi}{1 - e^{-\frac{\pi}{3\tan\varphi}}} e^{-\frac{\omega t - \pi/3}{3\tan\varphi}} + \sin(\omega t - \varphi) - \frac{\sin\theta}{\cos\varphi}\right] \tag{12.19}$$

Exercise 12.2

Fill in the blank(s) with the appropriate word(s).

i) Three phase full wave uncontrolled rectifier uses _________ diodes.

ii) Three phase full wave uncontrolled rectifier does not require ________ wire connection.

iii) In a three phase full wave uncontrolled rectifier each diode conducts for _______ radians.

iv) The minimum frequency of the output voltage ripple in a three phase full wave rectifier is _________ times the input voltage frequency.

v) The input ac line current of a three phase full wave uncontrolled rectifiers supplying an R – L – E load contain only ________ harmonics but no ________ harmonic or __________ component.

vi) A three phase full wave uncontrolled rectifier supplying an R – L – E load normally operates in the ________ conduction mode.

Answers: (i) six; (ii) neutral; (iii) 2π/3; (iv) six; (v) odd, tripler, dc; (vi) continuous.

2. A 220 V, 1500 rpm 20 A separately excited dc motor has armature resistance of 1Ω and negligible armature inductance. The motor is supplied from a three phase full wave uncontrolled rectifier connected to a 220 V, 3 phase, 50 Hz supply through a Δ/Y transformer. Find out the transformer turns ratio so that the converter applies rated voltage to the motor. What is the maximum torque as a percentage of the rated torque the motor will be able to supply without over heating. Assume ideal transformer and continuous conduction.

Answer: Average output voltage of the converter is

$$V_0 = \frac{3\sqrt{2}}{\pi} V_L = 220V$$

∴ V_L = 163 Volts. This is the line voltage of the secondary side of the transformer. The secondary is star connected. So

Secondary phase voltage $= \frac{163}{\sqrt{3}} = 94$ volts .

Primary side is delta connected. So

Primary phase voltage = 220 V.

∴ The required turns ratio = $\frac{220}{94} = 2.34:1$

Output voltage can be written as

$$v_0 = V_0 + \sum_{n=1}^{\alpha} v_{hn}$$ Where v_{hn} = nth harmonic voltage magnitude.

$$\therefore \qquad i_0 = \frac{V_0 - E}{r} + \sum_{n=1}^{\alpha} \frac{v_{hn}}{r}$$

Where E = back emf and r = armature resistance

$$\therefore \qquad I^2_{0RMS} = \left(\frac{V_0 - E}{r}\right)^2 + \sum_{n=1}^{\alpha} \left(\frac{V_{hn}}{r}\right)^2$$

$$= I^2_{0AV} - \frac{V^2_{0AV}}{r^2} + \frac{V^2_{0AV}}{r^2} + \sum_{n=1}^{\alpha} \left(\frac{V_{hn}}{r}\right)^2$$

$$= I^2_{0AV} - \frac{V^2_{0AV}}{r^2} + \frac{V^2_{0RMS}}{r^2}$$

$$= I_{0AV}^2 + \frac{V_{0AV}^2}{r^2}\left(FF^2 - 1\right)$$

To prevent over heating $I_{0RMS} = 20$ A

For the given converter $FF^2 = \left(\frac{V_{0RMS}}{V_{0AV}}\right)^2 = 1.00176$

$$\therefore \quad 20^2 = I_{0AV}^2 + \frac{220^2}{1}(1.00176 - 1)$$

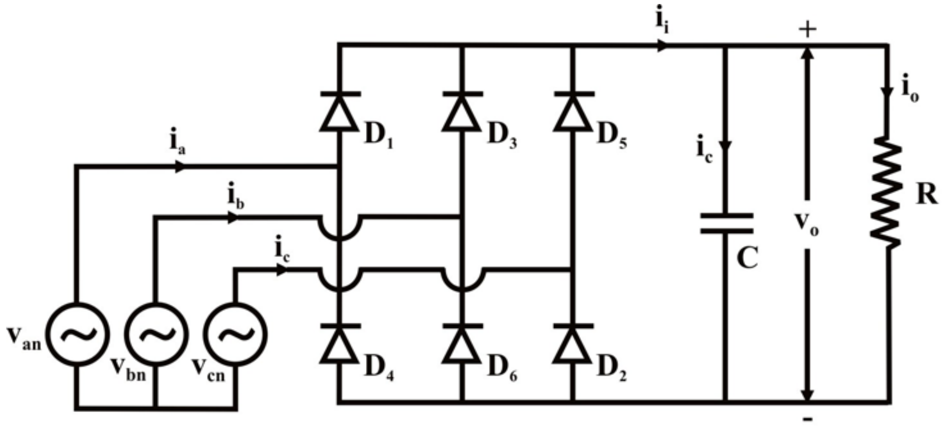
$$\therefore \quad I_{0AV}^2 = 314.816 \qquad \therefore \quad I_{0AV} = 17.743 \text{ Amps.}$$

In a separately excited dc machine $T_e \propto I_{0AV}$

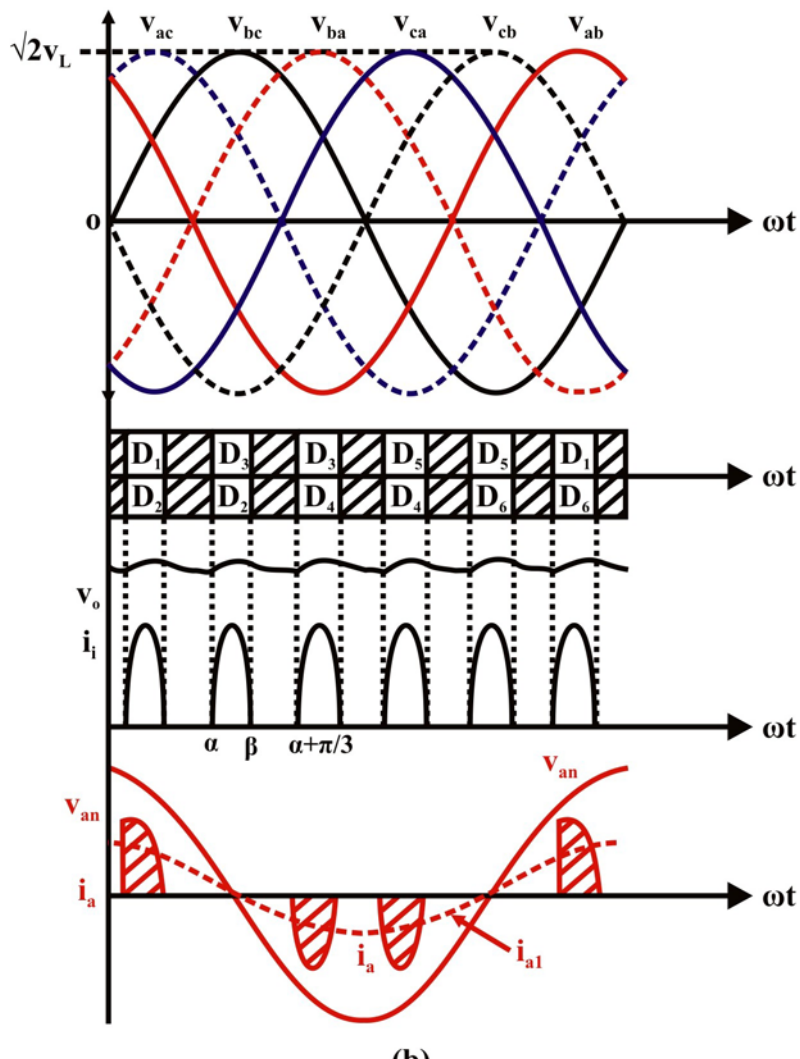
$$\therefore \text{ Maximum allowable torque} = \frac{17.743}{20} \times 100 = 88.715 \text{ \% of full load torque.}$$

12.3.2 Operation of a three phase uncontrolled bridge rectifier supplying a capacitive load

A three phase uncontrolled bridge rectifier supplying a capacitive load is a very popular power electronic converter. It is very widely used as the front end of a variable voltage variable frequency dc – ac inverter. Fig. 12.3 (a) shows the power circuit diagram of such a converter. Operation of the converter can be explained as follows. The top group diodes (D_1, D_3, D_5) form a "Maximum value circuit" and therefore the maximum of the phase voltages v_{an}, v_{bn}, v_{cn} appears at the positive dc bus. On the other hand, the bottom group diodes (D_2, D_4, D_6) form a "Minimum value circuit". Therefore the minimum of the phase voltages v_{an}, v_{bn} and v_{cn} appears at the negative dc bus. Therefore, the output voltage waveform at any instant is equal to the maximum of the six line voltages v_{ab}, v_{bc}, v_{ca}, v_{ba}, v_{cb} and v_{ac} provided at least one diode from the top group and one from the bottom group conducts at that instant. None of the diodes will conducts, however if the output capacitor voltage is larger than the maximum line voltage. All the six operating modes of a 3 phase bridge rectifier namely, D_1D_2, D_2D_3, D_3D_4, D_4D_5, D_5D_6 and D_6D_1 appear in that order. In addition an additional operating mode in which none of the diodes conduct appears in the conduction diagram as shown in Fig. 12.3 (b). During these periods the output capacitor discharges through the load. As the capacitor voltage decreases its voltage becomes equal to the incoming line voltage. At this instant the appropriate diodes from both the top and the bottom group starts conducting and continuous to do so till the sum of the capacitor charging current and the load current becomes zero.



(a)



(b)

Fig. 12.3: Operation of the 3-phase uncontrolled bridge rectifier supplying capacitive load.
(a) circuit diagram
(b) waveforms.

From Fig. 12.3 (b)
In the interval $\alpha \le \omega t \le \beta$

$$v_o = \sqrt{2}V_L \sin \omega t \tag{12.20}$$

$$\therefore \quad i_c = c\frac{dv_o}{dt} = \sqrt{2}V_L \ \omega c \cos \omega t \tag{12.21}$$

$$i_o = \frac{v_o}{R} = \sqrt{2}\frac{V_L}{R} \ \sin \omega t \tag{12.22}$$

$$\therefore \quad i_i = i_o + i_c = \sqrt{2}\frac{V_L}{R}\left[\omega RC \cos \omega t + \sin \omega t\right]$$

$$= \sqrt{2}\frac{V_L}{R}\sqrt{1+\omega^2 R^2 C^2} \ \cos(\omega t - \varphi) \tag{12.23}$$

$$\text{Where } \tan\varphi = \frac{1}{\omega RC}$$

At $\omega t = \beta$, $i_i = 0$

$$\therefore \quad \cos(\beta - \varphi) = 0 \qquad \text{or} \quad \beta = \frac{\pi}{2} + \varphi \tag{12.24}$$

in the interval
$\beta \le \omega t \le \alpha + \pi/3$

$$c\frac{dv_o}{dt} + \frac{v_o}{R} = 0$$

$$v_o\big|_\beta = \sqrt{2}V_L \sin\beta = \sqrt{2}V_L \cos\varphi = \sqrt{2}V_L \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}} \tag{12.25}$$

$$\therefore \quad v_o = v_o\big|_\beta \ e^{-\frac{(\omega t - \beta)}{\omega RC}} = \sqrt{2}V_L \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}} e^{-\frac{(\omega t - \beta)}{\omega RC}} \tag{12.26}$$

at $\omega t = \alpha + \pi/3$

$$v_o = \sqrt{2}V_L \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}} e^{\frac{\pi/6 - \alpha + \varphi}{\omega RC}} \tag{12.27}$$

Also at $\omega t = \alpha + \pi/3$

$$v_o = \sqrt{2}V_L \sin\left(\omega t - \frac{\pi}{3}\right)\Bigg|_{\omega t = \alpha + \frac{\pi}{3}}$$

$$= \sqrt{2}V_L \sin\alpha$$

$$\therefore \quad \sin\alpha = \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} e^{\frac{\pi/6 - \alpha + \tan^{-1}\frac{1}{\omega RC}}{\omega RC}} \quad (12.28)$$

From which the value of α can be found. Equation 12.23 gives the expression of the output current i_i of the rectifier.

It is observed that i_i is discontinuous and contains large ripple. This is a major disadvantage of this converter. This ripple is also reflected in the input current of the rectifier as shown in Fig 12.3 (b). However, the displacement factor of the converter still remains unity.

The current i_i can be made continuous by connecting an inductor of appropriate value between the rectifier and the capacitor. Analysis of such a converter is similar to a converter supplying an R – L – E load where the value of E is $3\sqrt{2}\frac{V_L}{\pi}$.

Exercise 12.3

Fill in the blank(s) with the appropriate word(s)

i) A three phase full wave uncontrolled rectifier supplying a capacitive load can operate in the _________ conduction mode.

ii) The output _________ ripple factor of a three phase full wave uncontrolled rectifier supplying a capacitive load is very low.

iii) The output _________ ripple factor of a three phase full wave uncontrolled rectifier supplying a capacitive load is very high.

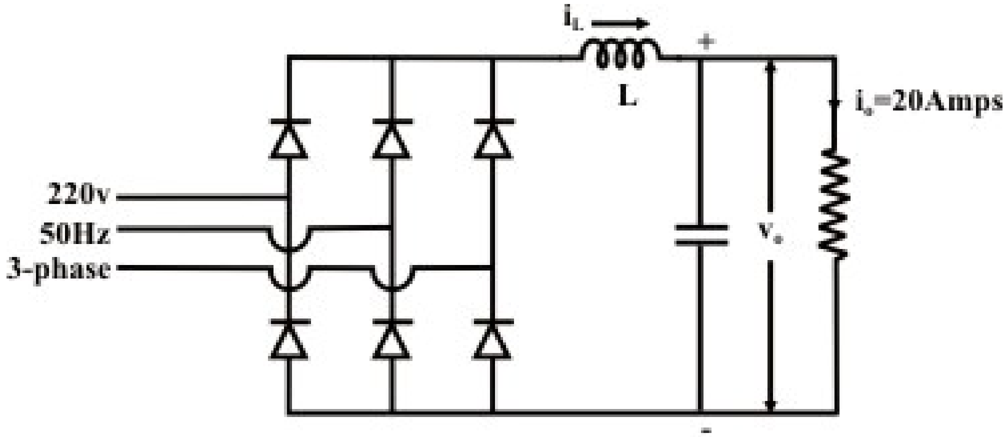
iv) The input current displacement factor of a three phase full wave uncontrolled rectifier supplying a capacitive load is ___________.

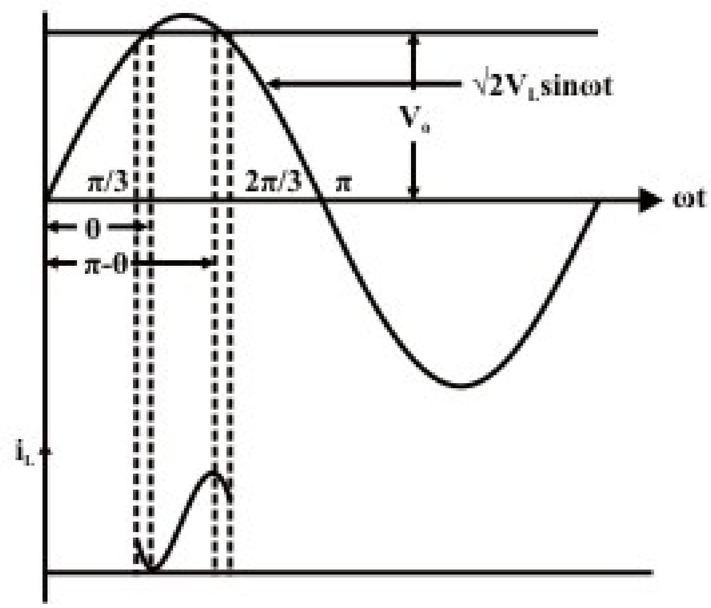
v) The input current distortion factor of a three phase full wave uncontrolled rectifier supplying a capacitive load is very ________.

Answers: (i) discontinuous; (ii) voltage; (iii) current; (iv) unity; (v) high.

2. A three phase full wave rectifier operates from 220 volts, three phase 50 Hz supply and supplies a capacitive resistive load of 20 Amps. An inductor of negligible resistance is inserted between the rectifier and the capacitor. Assuming the capacitor to be large enough so that the output voltage is almost ripple free. Calculate the value of the inductor so that the rectifier output current is continuous.

Answers: The following figure shows the circuit arrangement and the corresponding waveforms.





Since the conduction is continuous

$$V_0 = \frac{3\sqrt{2}}{\pi}V_L \quad \text{and} \quad \sin\theta = \frac{V_0}{\sqrt{2}V_L} = \frac{3}{\pi} \quad \text{or} \quad \theta = 72.73^o$$

In the interval $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$

$$v_0 + L\frac{di_L}{dt} = \sqrt{2}V_L \sin\omega t$$

Since v_0 is almost ripple free $v_0 = V_0 = \frac{3\sqrt{2}}{\pi}V_L$

$$\therefore \quad \frac{3\sqrt{2}}{\pi}V_L + \omega L\frac{di_L}{d\omega t} = \sqrt{2}V_L \sin\omega t$$

$$i_L = I_0 - \frac{\sqrt{2}V_L}{\omega L}\cos\omega t - \frac{3\sqrt{2}}{\pi\omega L}V_L\, \omega t$$

Now $\left. i_L \right|_{av} = 20A$

$$\therefore \qquad I_0 - \frac{3\sqrt{2}}{\pi\omega L} V_L \times \frac{1}{2} \times \frac{\pi^2}{3} \times \frac{3}{\pi} = 20A \text{ or } I_0 = 20 + \frac{3V_L}{\sqrt{2}\omega L}$$

$$\therefore \qquad i_L = 20 + \frac{\sqrt{2}V_L}{\omega L}\left[\frac{3}{2} - \cos\omega t - \frac{3}{\pi}\omega t\right]$$

For just continuous conduction $i_L = 0$ at $\omega t = \theta$

$$\therefore \qquad 0 = 20 + \frac{\sqrt{2}V_L}{\omega L}\left[\frac{3}{2} - \cos\theta - \frac{3}{\pi}\theta\right]$$

or $\omega L = 1.0306\ \Omega$ or $L = 3.28$ mH.

References

[1] "Power Electronics", P.C. Sen; Tata MC Grawhill publishing company limited; 1995.

[2] "Power Electronics, Converters, Applications and Design"; Mohan, Undeland, Robbins; John Willey and Sons Ine, Third Edition, 2003.

Lesson Summary

- Three phase uncontrolled rectifiers are available in half wave and full wave configuration.
- Three phase uncontrolled half wave rectifier require three phase four wire power supply.
- The input ac line current in a three phase uncontrolled half wave rectifier contain dc component which may cause "dc saturation" of input transformer.
- Three phase full wave uncontrolled rectifier is most widely used in the medium power applications particularly as the input stage of the dc link inverter.
- Three phase full wave uncontrolled rectifier uses six diodes instead of three of the half wave rectifier.
- Full bridge rectifier does not require neutral connection.
- The output voltage of a three phase full bridge rectifier contains multiplies of 6^{th} harmonic of input cycle.
- The input ac current of a three phase full bridge rectifier contain only odd harmonics but no dc component or triplen harmonics.
- The input displacement factor of the three phase bridge rectifier is always unity.
- Three phase full bridge converter supplying an R – L – E load usually operate in the continuous conduction mode.
- Compared to single phase rectifiers, three phase bridge converter require smaller inductor to obtain the same output current ripple factor.

- Three phase bridge rectifier supplying a capacitive load has very good output voltage form factor but very poor input current THD.
- Compared to single phase converters three phase bridge rectifier require smaller capacitor to obtain a given output voltage form factor.

Practice Problems and Answers

Q1. A three phase half wave rectifier operates from a three phase 220 V, 50 Hz supply and supplies a resistive load rated at 200 Volts 1 KW through an inductance large enough to make the load current ripple free. Find out the power consumed by the load? What will be the load power if the inductor is shorted?

Q2. A three phase full wave rectifier operates from a three phase 220 V 50 Hz supply through a three phase Δ/Y transformer and supplies a 200 V 1500 R.P.M, 50 Amps separately excited dc motor. Find out the turns ratio of the transformer so that the motor operates at rated speed at full load. If the motor armature resistance is 0.5 Ω find out the inductance to be connected in series with the motor such that the rectifier operates in the continuous conduction mode at 50 % of the full load torque.

Q3. A three phase full wave rectifier supplies a resistive capacitive load of 50 Amps from a 220 V. 3 phase 50 Hz supply. Find out the value of the load capacitance such that the load voltage ripple is less than 5 %.

Answers to practice problems

1. Since the load current is ripple free the power consumed by the load will be

$$P_L = I_{0AV}^2 R_{LOAD} = \frac{V_{0AV}^2}{R_{LOAD}}$$

Now $V_{0AV} = \frac{3\sqrt{2}V_L}{2\pi} = \frac{3\sqrt{2}\times 220}{2\pi} = 148.55 \text{ volts}$

$\therefore \quad P_L = \frac{V_{0AV}^2}{R_{LOAD}} = \left(\frac{148.55}{200}\right)^2 \times 1\,\text{KW} = 551.7 \text{ watts}$

When the inductor is shorted

$$P_L = \frac{V_{0RMS}^2}{R_{LOAD}}$$

Now from Equ. (12.2) $V_{0RMS} = \sqrt{\frac{1}{3} + \frac{\sqrt{3}}{4\pi}}\, V_L = 151.01 \text{ volts}$

$$\therefore \qquad P_L = \frac{V_{0RMS}^2}{R_{LOAD}} = \left(\frac{151.01}{200}\right)^2 \times 1\,\text{KW} = 570 \text{ watts}$$

2. To run at rated speed at full load the motor terminal voltage must be 200 volts.

$$\therefore \qquad V_{0AL} = \frac{3\sqrt{2}}{\pi} V_L = 200 \text{ volts}, \ \therefore \ V_L = 148.1 \text{ volts}$$

Where V_L is the secondary line voltage. Secondary is star connected. So secondary phase voltage

$$V_2 = \frac{V_L}{\sqrt{3}} = 85.5 \text{ volts}$$

Primary is delta connected. So primary phase voltage

$V_1 = 220$ V

$\therefore$ Required turns ratio = $\dfrac{V_1}{\sqrt{2}} = 1 : 0.38865$

At 50% of the full load torque motor current is 25 Amps

$\therefore$ back Emf = 200 – 0.5 × 25 = 187.5 Volts.

$\therefore$ speed at 50% of full load torque = $\dfrac{187.5}{200 - 0.5 \times 50} \times 1500 = 1607 \text{ RPM}$.

At 50% of full load torque the motor operates in the continuous conduction mode, with reference to Fig. 12.2 and equation 12.19.

$$i_0 = \frac{\sqrt{2}V_L}{z}\left[\frac{\sin\varphi}{1 - e^{-\frac{\pi}{3\tan\varphi}}} e^{-\frac{\omega t - \pi/3}{\tan\varphi}} + \sin(\omega t - \varphi) - \frac{\sin\theta}{\cos\varphi}\right]$$

Where $\sin\theta = \dfrac{E}{\sqrt{2}V_L} = \dfrac{187.5}{200} = 0.9375$

$\theta = 69.64^o = 1.2154$ rad.

At the junction of continuous and discontinuous conduction

$$i_0\big|_{Min} = i_0\big|_{\omega t = \theta} = 0$$

$$\therefore \qquad \frac{\sin\varphi}{1 - e^{-\frac{\pi}{3\tan\varphi}}} e^{-\frac{(\theta - \pi/3)}{\tan\varphi}} + \sin(\theta - \varphi) - \frac{\sin\theta}{\cos\varphi} = 0$$

OR $$\frac{1}{2}\frac{\sin 2\varphi e^{\frac{(\pi/3 - \theta)}{\tan\varphi}}}{1 - e^{-\frac{\pi}{3\tan\varphi}}} + \frac{1}{2}\sin\theta - \frac{1}{2}\sin(\theta - 2\varphi) = \sin\theta$$

OR $$\sin 2\varphi \frac{e^{\frac{(\pi/3-\theta)}{\tan\varphi}}}{1-e^{-\frac{\pi}{3\tan\varphi}}} - \sin(\theta-2\varphi) = \sin\theta$$

Solving which $\varphi = 34.65^o$.

$$\therefore \quad \frac{\omega L}{R} = \tan\varphi = 0.6911$$

$\therefore \quad \omega L = 0.3456\ \Omega \qquad$ or $L = 1.1$ mH.

3. Assuming linear ripple

$$V_{0AV} = \frac{V_{0Max} + V_{0Min}}{2}$$

$$\hat{V}_{0pp} = V_{0Max} - V_{0Min}$$

$$\therefore \quad \frac{2\left(V_{0Max} + V_{0Min}\right)}{V_{0Max} + V_{0Min}} = \frac{\hat{V}_{0pp}}{V_{0AV}} = 0.05$$

$$\therefore \quad \frac{1 - V_{0Min}/V_{0Max}}{1 + V_{0Min}/V_{0Max}} = 0.025$$

$$\therefore \quad V_{0Min}/V_{0Max} = 0.9512\ .$$

From Fig. 12.3, $\qquad V_{0Max} = \sqrt{2}V_L = \sqrt{2}\times 220\text{ V} = 311$ volts

$\therefore \quad V_{0Min} = 295.943$ Volts $\qquad \therefore \ V_{0AV} = 303.47$ V.

$I_{0AV} = 50$ Amps $\qquad \therefore \ R = 6.0694\ \Omega$.

From Fig. 12.3. $\qquad V_{0Min}$ occurs at $\omega t = \alpha$

$\therefore \quad V_{0Min} = \sqrt{2}V_L \sin\alpha = 295.943$

$\therefore \quad \sin\alpha = 0.9512 \qquad$ or $\alpha = 72^o$

But from Equation (12.28)

$$\sin\alpha = \cos\varphi\, e^{\tan\varphi\left(\frac{\pi}{6} - \alpha + \varphi\right)}$$

where $\tan\varphi = \dfrac{1}{\omega RC}$

from which $\varphi = 3.5^o \quad \therefore \ \tan\varphi = 0.06116$

$\therefore \quad \omega RC = \dfrac{1}{\tan\varphi} = 16.35\ , \qquad R = 6.0694\ \Omega$

$\therefore \quad \omega C = 2.6938, \qquad \therefore \ C = 8575\ \mu F$.