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Chapter one

Instrumentation & Measurement

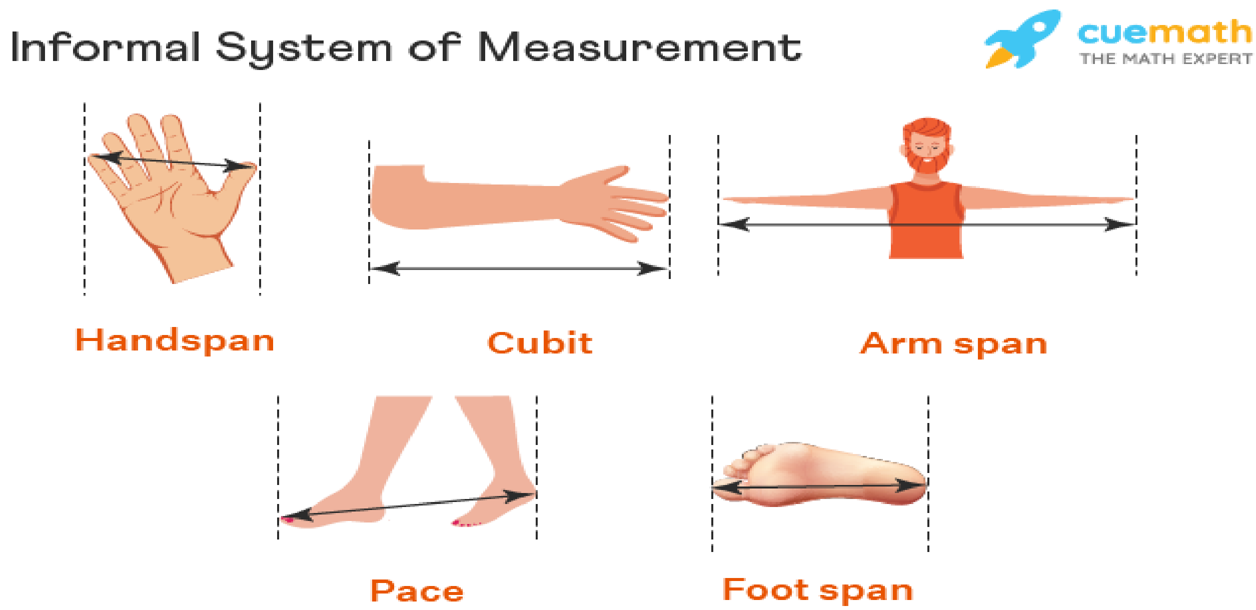
System of Measurement

System of Measurement

System of measurement refers to the process of associating numbers with physical quantities and phenomena. It is more like a collection of units of measurement and rules relating them to each other. The whole world revolves around measuring things! Everything is measured: the milk you buy, the gas you fill for the vehicle, the steps you walk. Even our productivity is measured in terms of productivity indexes on how productively we work. System of measurement is very important and define and express the different quantities of length, area, volume, weight, in our day-to-day communications. The system of measurement is based on two important foundation pillars of defining the basic unit of measurement, and the measure of conversion from the basic unit to other related units

What is System of Measurement?

Measurement systems are a collection of units of measurement and rules relating them to each other. The word “measurement” is derived from the Greek word "metron," which means a limited proportion. This word also finds its roots in the words "moon" and "month", possibly because astronomical objects were among the first methods to measure time. In the old days, we used body parts for informal measurement systems like foot length, cubit, and hand span, etc. which were not so accurate and vary from person to person.



So, there was a need to regularize the measurements. A system of measurement like the International System of Units called the SI units (the modern form of the metric system), Imperial system, and US customary units were standardized across the world.

Introduction to Metric System of Measurement

A metric system is a system of measurement based on the standard units as a meter for length, kilogram for mass, and liter for volume. It was introduced in France in the 1790s and is now being used officially by many countries around the world. The metric system is based on the international decimal system. The base units used in the metric system are used to derive higher and lower units of measurement. Often the required unit is either larger or much small than the defined units. Let us now look at the below described, different systems of measurement.

Metric System: The units of the metric system, originally taken from observable features of nature (basically what we normally measure like the

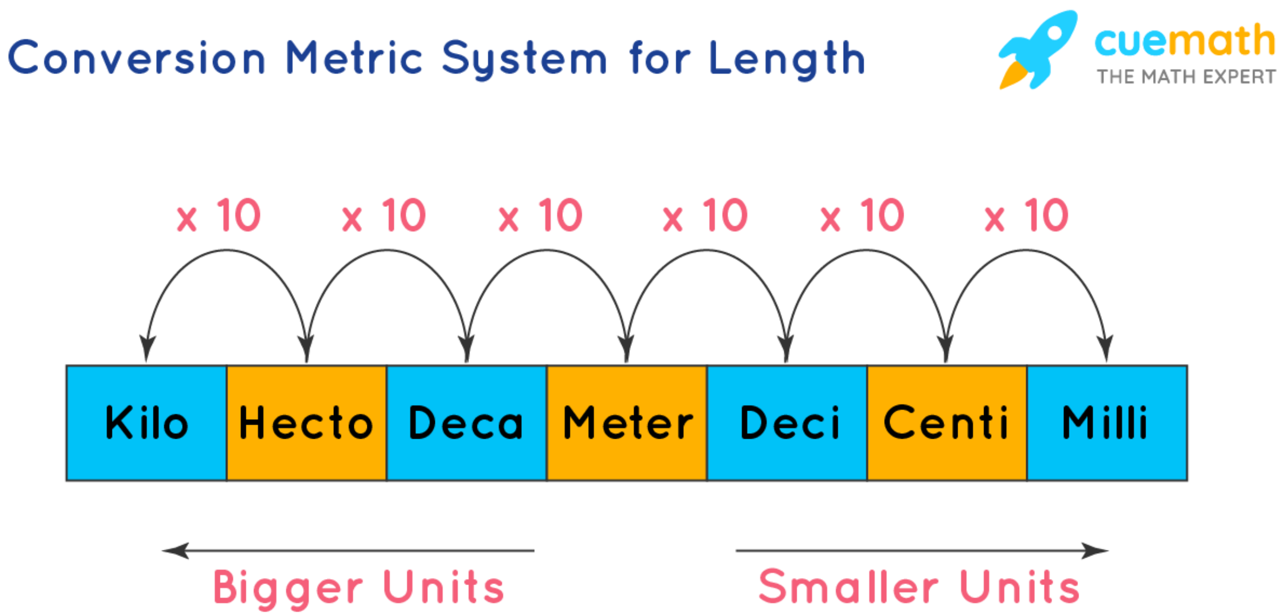
time, length, mass, etc. are defined by seven physical constants with numerical values in terms of the units. Metrics systems evolved and over time are universally accepted as the International System of Units called the SI System. Many countries follow this system.

US Standard Units: United States, Liberia, and Myanmar have not adopted the metric system as their official system of weights and measures. U.S. customary units are used across the states for measurements.

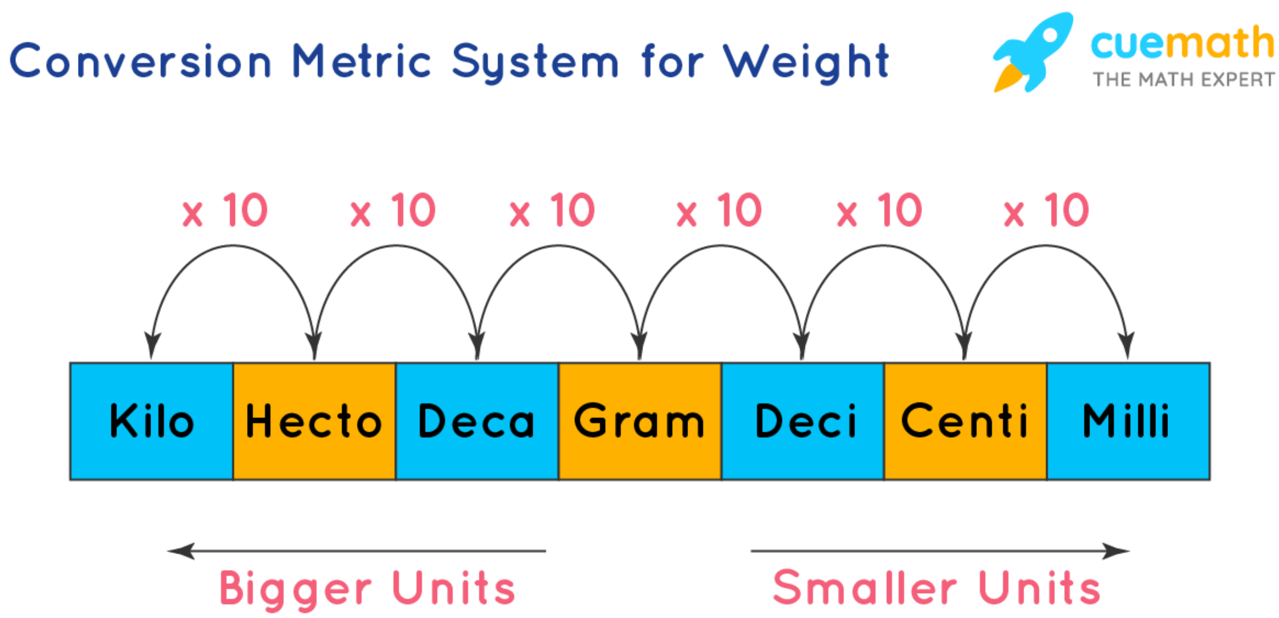
Metric Systems of Measurement

The Metric system has 3 main units namely, meter to measure the length, kilogram to measure the mass, and seconds to measure time.

Meter: Length is measured in meters. The unit is denoted by the alphabet (m). Look at the chart below. The base unit is "m" and we add "Deca," "Hecto," and "Kilo" to measure large units by successively multiplying by 10 and "deci," "centi," and "millie" successively dividing by 10, to measure smaller length. We can use a simple ruler to measure length. For example, a pencil measured on a ruler would be 10 cm long.



Kilogram: Mass is measured in kilograms and the unit is denoted by (kg). It tells us how heavy or how light an object is. We can multiply and divide the base units to measure smaller and bigger units. In general, for our convenience, we use gram, kilogram, and milligram. Other units are hardly used. We use a weighing scale to measure how heavy things are. A weighing scale is used in supermarkets to weigh groceries. A doctor also used a weighing scale to find the weight of a person.

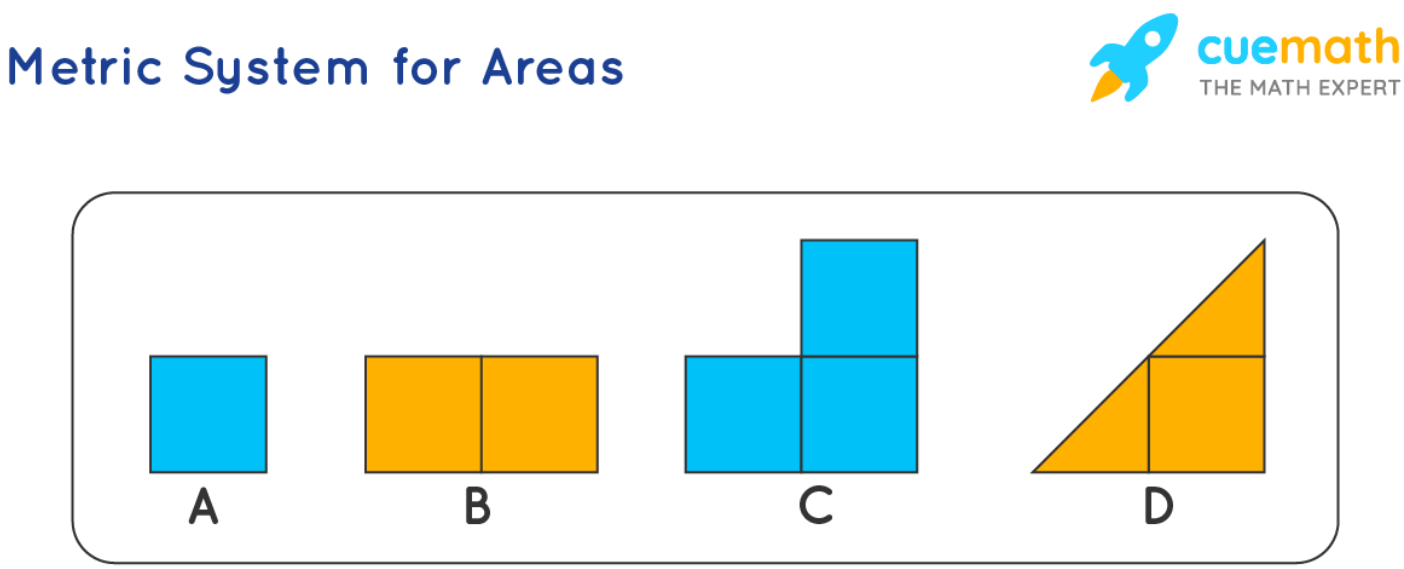


Second: Time is measured in seconds. The representation of seconds is (s). Time is the ongoing sequence of events taking place. It is used to quantify the duration of the events. It also helps us to set the start time or the end time of events. The base unit for time is seconds. Some of the conversion units of time are, 1 minute = 60 seconds, 1 hour = 60 minutes, 1 day = 24 hours, 1 week = 7 days, 1 year = 12 months, or 1 year = 365 days. We use a clock or watch to tell the current time. A stopwatch can be used to measure the time in seconds.

Other Metric System of Measurements

Though we are aware of the basic defined metric systems for length, mass, volume, but there are numerous other quantities in the physical world, for which we need to define the base unit. Quantities like, force, power, area, magnetic intensity, have their own individual units, which have been derived from the basic 7 quantities of the metric system of measurement.

Area: The area is the space occupied by a two-dimensional shape or figure. The area is measured in square units like sq. cm or cm^2, sq. m or m^2, sq km or km^2, etc. Let us look at the below example. If the area of each square is 1 cm^2. The area of shape A = 1cm^2. Area of shape B = (1+1) = 2 cm^2. Area of shape C = (1+1+1) = 3 cm^2. Area of shape D = (0.5 +1 +0.5) = 2 cm^2. Now that you know what an Area is, let's learn how to find the Area of a Triangle, Area of a Quadrilateral, Area of a Circle.



Volume: Volume is the space enclosed or occupied by any three-dimensional object or solid shape. It has length, width, and height. It is measured in cubic units like cm^3, m^3, etc. and liquid volume is measured in liters. Let us look at a simple example. The initial volume of water in the container is 20 units. The volume of water when the object is placed inside the container 30 is units. Therefore, the Volume of the object is the difference between the two volumes, that is, 30-20 = 10 units. Finding the Volume of an object can help us determine the amount required to fill that object, for example, the amount of

water in a bottle. Now, let's learn how to find the Volume of a Cuboid, Volume of a Cylinder, Volume of a Cone, Volume of a Sphere.

Time: Time is the ongoing sequence of events taking place. It is used to quantify the duration of the events. It also helps us set the start time or the end time of events. One of the very first experiences we have with mathematics is learning how to measure time. You may already know that the measurement of time is done using a watch and a calendar. Now, let's learn how to read and represent time along with how to read a calendar.

Speed: Speed is the change in position of the object with respect to time. It is the ratio of distance traveled by the object to the time taken to travel that distance. The SI unit of speed is m/s. It is also expressed in km/hr or miles/hr. In vehicles, we have a speedometer that records the speed at which the vehicle is traveling.

Acceleration: Acceleration is the rate of change of velocity with respect to time. It is a vector quantity. It has magnitude and direction. It is measured in meters per second^2 (m\s^2), and the dimension is LT^{-2}.

Force: Force is a push or a pull. When two objects interact with each other, the object changes its position based on the force acting on it. You apply force to move an object from its place and you also apply force to stop a moving object. The SI unit of force is Newton named after the scientist named Newton. It is basically kgm/s^2 (kilogram - meter per second^2), and the dimension is LMT^{-2}.

SI Units: The international system of units called the SI units is derived from the metrics system. The basic 7 measurable quantities are standardized, and they use the units listed below in the table. There are the basic 7 units of

measurement, and the rest other units are derived from here like the area, volume, force, acceleration, etc we just discussed above. Please find below the seven different quantities and their units of measurements.

Quantity	**SI Units**
Length	**meter**
Time	**seconds**
Temperature	**kelvin**
Electric Current	**ampere**
Luminous intensity	**candela**
Mass	**kilogram**

US Standard System of Measurements

Just like the metrics system, the US follows the imperial system of units, also called the U.S customary units. Here things are measured in feet, inches, pounds, ounces, etc. Let us explore them in detail in the following sections.

Length: The four most commonly used measures of lengths are inch, feet, yards, miles. Let us look at the conversions from one unit to another. 1 inch = 2.54 cms. 1 foot = 12 inches. 1 yard = 3ft or 36 inches. 1 mile = 1760 yards(5280 ft), (1 metrics 1.609344 km.)

Area: An area is a two-dimensional unit. It is the amount of space occupied by the object. We use inches, feet, yards, miles to measure the length and thus

area too. The area is measured in square units such as square inch, square foot, square yard, square mile, acre. A small area is measured in a square inch and larger surfaces are measured in square yards. Land area is usually measured in acres. Let us look at a few examples of areas. A chessboard is 100sq inches, a garage is 200 sq ft, a part is 100 sq yards, a botanical garden is 500 sq miles, a football ground is exactly 1 acre (1 acre = 43,560 feet.)

Volume: Volume is a three-dimensional quantity. It is the amount of capacity/space a substance contains, or the space it can hold. The most common measures of volume in the US customary units are fluid ounces (fl. oz), cups, pints, quarts, and gallons. Note that an ounce is the measure of mass and a fluid ounce is a measure of volume. A fluid ounce is the size of a medicine cup. Other units like peck (1 peck = 2 gals), barrel (31.5 gals) are hardly used. Let us look at the conversion from one unit to another. 1 cup = 8 fl oz. 1 pint = 2 cups. 1 quart = 2 pints, 1 gallon = 4 quarts.

Mass: The most common measurements of mass in the US customary units are ounces (oz), pounds (lb), and tons (ton). Other very small units like dram (weight of grain) are hardly used. Let us look at the conversions from one unit to another. 1 ounce = 16 dr, 1 pound = 16 oz, and 1 ton = 2000 lb. There are two variants for the ton. A short ton is 2,000 pounds, and a long ton is 2,240 pounds. In general, when we say a ton, it means a short ton.

Time and Date: Time is measured in seconds. We use a clock (digital or analog) to tell the current time. There are 12-hr clock and 24-hour clock formats as well. Further, let us learn more about measuring time, reading and representing time, and how to read a calendar. In the US, the format for writing the date is "month–day–year". For example 7/4/2000 means the 7th

month - July, 4 - Date, and the year 2000. It is the Millenium year Independence Day, July 4th!

Temperature: Temperature is the measure of how hot or cold substances are. We use a thermometer to measure the temperature. The temperature is measured in degrees Celsius (°C) or Fahrenheit (°F). 0 degrees celsius is equal to 32 degrees Fahrenheit. To convert Celsius to Fahrenheit, you can use the formula Fahrenheit = 9/5 × Celcius + 32

Speed: Speed is the total distance covered by an object in a given time. It is the ratio of distance covered to the time traveled. In British and US customary units, speed is measured in miles per hour (mph). The speedometer is an instrument that gives the current speed at which the vehicle is traveling. You might have seen it in your car.

Direction: The four cardinal directions are North, South, East, and West. A magnetic compass tells the direction. The diagonal directions include the North-East, North-West, South-East, and South-West. A compass is generally used for navigation and is used in seas, deserts, where it is difficult to find the direction.

Conversions from One System of Measurement to Another

The conversion of length, mass, area, volume from one system of measurement to another system of measurement is very helpful to identify the units. In the below set of tables, we have listed the different units of length, area, volume, mass from US standard measurement to the metric measurement system. Please find below the different units of conversion of length from US standard measurement to the metric measurement system.

Length Conversions	
US standard meaurement	Metric Measurement
1 inch	2.54 cm
1 ft	0.3048 m
1 yard	0.914 m
1 mile	1.609 km

Please find below the different units of conversion of mass from US standard measurement to the metric measurement system.

Mass Conversions	
US Standard Measurement	Metric Measurement
1 ounce	28.34g
1 pound	0.453 kg
1 ton	907.184 kg

The following table shows the different units of conversion of the volume from US standard measurement to the metric measurement system.

Volume(capacity) Conversions

US standard measurement	Metric measurement
1 fluid ounce	29.573 ml
1 fluid pint	0.473 l
1 fluid quart	0.946 l
1 gallon	3.785 l

Please find below the different units of conversion of the area from US standard measurement to the metric measurement system.

Area Conversions	
US standard measurement	Metrics measurement
1 sq.in	6.45 sq cm
1 sq ft	0.0929 sq.m
1 sq yd	0.836 sq m
1 acre	4046.86 sq m

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- Metric Conversion Chart
- Unit Conversion

- Convert km/h to m/s
- Length Conversion

System of Measurement Examples

- **Example 1: Using the system of measurement, determine how many feet are there in 6 miles?**

Solution:

From the conversion chart above, we can see that 1 mile = 5280 ft. Thus, in 6 miles there are 6 × 5280 = 31,680 ft. Therefore, 6 miles = 31,680 feet.

- **Example 2: Emy measured 3 inches in her inch scale. How many centimeters will it be approximately equal to?**

Solution:

Using system of measurement, we have 1 in = 2.54 cm. Thus, 3 in = 3 × 2.54 = 7.62 cm. Therefore, 3 inches = 7.62 cm.

- **Example 3: Which of the following weighs the heaviest? (a) a sack of wheat weighing 12 kg (b) a bag of rice weighing 0.15 tons (c) a box of corn weighing 100 pounds. Use system of measurement to find the answer.**

Solution:

We know that 1 pound is roughly 1/2 kg. Hence 100 pounds is approximately 50 kg. 1 ton is roughly 1000 kg. 0.15 tons is approximately 150 kg. So, a bag of rice weighing 0.15 tons weighs the heaviest. Therefore, a bag of rice of 0.15 tons weighs the heaviest.

- **Example 4: A football ground exactly measures 1.32 acres in size. Using the system of measurement, determine how many square feet does it measure?**

Solution:

We know that,1 acre = 43560 sq ft. Thus, 1.32 acre = 1.32 × 43560 = 57499.2 sq. ft. Therefore, 1.32 acres = 57499.2 sq.ft

- **Example 5: Water boils at 100 degrees Celsius. What is the temperature in degrees Fahrenheit?**

Solution:

As per the measurement system, Fahrenheit = 9/5 × C +32. Substituting C = 100, we get, F = 9/5 ×100 +32. Simplifying it further, we get, F = 9 × 20 +32 = 180 + 32 = 212.F. Therefore, 100 Celsius is equal to 212 Fahrenheit.

FAQs on System of Measurement

Why Systems of Measurements Are Used in the Metric System?

Measurement is important in our day-to-day life to get the right amount of quantity we require. Exact measurement helps in taking correct decisions. The basic units of measures of the metric system are length (measured in meter) mass (measured in kilogram) and time (measured in seconds). Meter, kilogram, and seconds are the base units, kilo-, hecto-, deka-, deci-, centi-, and milli- are prefixed with a base unit to measure larger or smaller quantities.

What Are the 7 Basic Units of Measurement?

The basic 7 units of measurement are listed below, the rest of the measures are derived from the base units.

Quantity	**SI Units**
Length	**meter**

Time	**seconds**
Temperature	**Kelvin**
Electric Current	**ampere**
Luminous intensity	**candela**
Mass	**kilogram**

What Are the 3 Systems of Measurements?

The three standard systems of measurements are the International System of Units (SI) units, the British Imperial System, and the US Customary System. Of these, the International System of Units(SI) units are prominently used.

What Is the Main System of Measurement?

The main system of measurement is the international system of units(SI) units, and all other systems of measurement are linked to it. The British Imperial system and the US customary system are linked to the SI units of measurement with the units of conversion and can be conveniently used to convert from one unit to another.

How To Read the Metric System of Measurement?

The metric system of units can be read with the basic units and their conversions. Length, weight, volume, time are the basic dimensions of the measurement of the metric system. The basic unit of length is the meter and all higher units are derived by multiplying the basic unit with an exponent of

10. Similarly, the basic unit of weight is grams, for the volume, it is liters, and for the time it is seconds.

What Is the Customary System of Measurement Based on?

The customary system of measurement refers to the US system of measurement. It is considered as the main system of measurement and its units. The system is based on the standard units as, yard for length, the pound for weight, the gallon for liquid volume, and the bushel for dry volume.

How Does the System of Measurement Work?

The system of measurement works on two basic principles. Firstly, the basic units of measurement for the system are to be defined for the various physical quantities. Secondly, the conversion of these units to higher and lower units should be defined, along with their conversion to other systems of measurements. With these two principles, the systems of measurement are completely defined and work smoothly.

Chapter Two
Direct Current Indicationg Instruments

Torque and Deflection of the dc Current Galvanometer

Although the suspension galvanometer is neither a practical nor portable instrument, the principles which govern its operation apply equally to its more modern version, the *permanent-magnet moving-coil mechanism* (PMMC). Figure (1 shows the construction of the PMMC mechanism. The different parts of the instrument are identified alongside the figure.

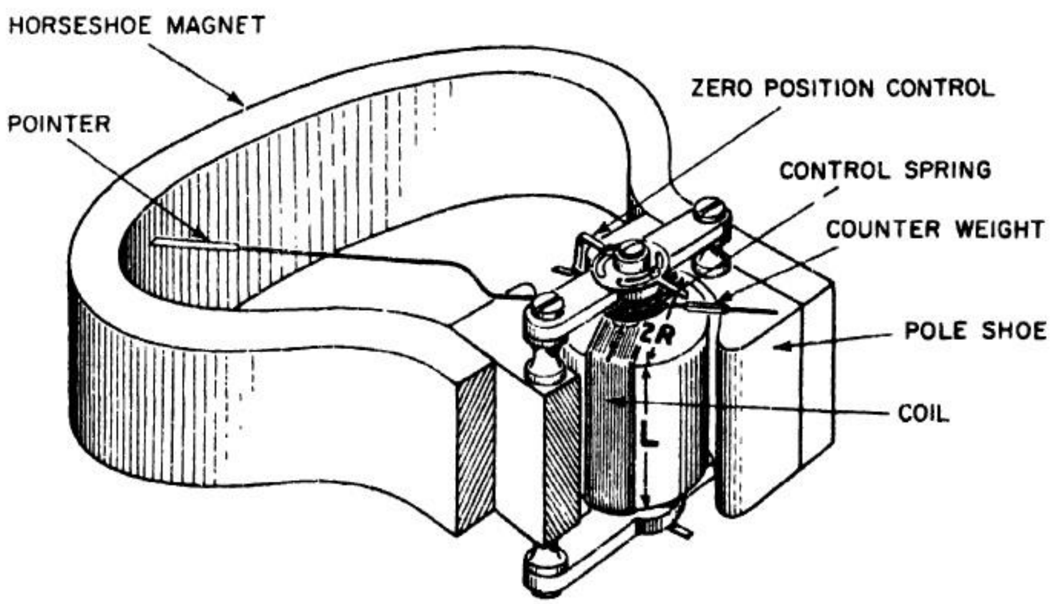


Fig (1)

Construction details of the external magnet PMMC movement (*Courtesy Weston Instruments, Inc.*).

Here again we have a coil, suspended in the magnetic field of a permanent magnet, this time in the shape of a horseshoe. The coil is suspended so that it can rotate freely in the magnetic field. When current flows in the coil, the developed electromagnetic (EM) torque causes the coil to rotate. The EM torque is counterbalanced by the mechanical torque of control springs attached to the movable coil. The balance of torques, and therefore the angular position of the movable coil, is indicated by a pointer against a fixed reference, called a *scale*.

The equation for the developed torque, derived from the basic law for electromagnetic torque, is

$$T = B \times A \times I \times N \qquad \text{........(1)}$$

where T = torque, newton-meter (N-m)
B = flux density in the air gap, webers/square meter (Wb/m²)
A = effective coil area, square meters (m²)
I = current in the movable coil, amperes (A)
N = turns of wire on the coil.

Equation (4-1) shows that the developed torque is directly proportional to the flux density of the field in which the coil rotates, the current in the coil, and the coil constants (area and turns). Since both flux density and coil area are *fixed* parameters for a given instrument, the developed torque is a direct indication of the current in the coil. The pointer deflection, therefore, can be used to measure the current. Equation (1) also shows that the designer may vary only the value of the control torque and the number of turns on the moving coil to measure a given full-scale current. The practical coil area generally ranges from approximately 0.5 to 2.5 cm^2. Flux densities for modern instruments usually range from 1,500 to 5,000 gauss (0.15 to 0.5 Wb/m^2). Thus, a wide choice of mechanisms is available to the designer to meet many different measurement applications.

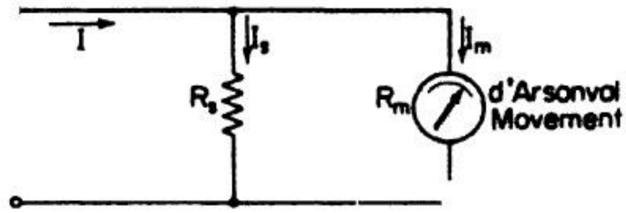
measurement applications.

A typical panel PMMC instrument, with a $3\frac{1}{2}$-in. case, a 1-mA range, and full-scale deflection of 100 degrees of arc, would have the following characteristics:*

$$A = 1.75 \text{ cm}^2$$
$$B = 2{,}000 \text{ G } (0.2 \text{ Wb/m}^2)$$
$$N = 84 \text{ turns}$$
$$T = 2.92 \times 10^{-6} \text{ N-m}$$
$$\text{Coil resistance} = 88\ \Omega$$
$$\text{Power dissipation} = 88\ \mu\text{W}$$

Ammeter Shunts and Multirange Ammeters

The *basic movement* of a *dc ammeter* is a PMMC d'Arsonval galvanometer. The coil winding of a basic movement is small and light, and can carry only very small currents. It is necessary to extend the current range of dc ammeters. When high currents are to be measured, the major part of the current bypasses the basic movement through a resistance, called a *shunt*. Figure (2) shows the basic movement and its shunt to produce an *ammeter*.



Figer(2)

The basic dc ammeter circuit.

The resistance of the shunt can be calculated using conventional circuit analysis. See Fig. (2)

where R_m = internal resistance of the movement (the coil)

R_s = resistance of the shunt

I_m = full-scale deflection current of the movement

I_s = shunt current

I = full-scale current of the ammeter including the shunt.

Since the shunt resistance is in parallel with the meter movement, the voltage drops across shunt and movement must be the same and we can write

$$V_{\text{shunt}} = V_{\text{movement}}$$

or

$$I_s R_s = I_m R_m \quad \text{and} \quad R_s = \frac{I_m R_m}{I_s} \qquad \text{..........(2)}$$

Since $I_s = I - I_m$, we can write

Since $I_s = I - I_m$, we can write

$$R_s = \frac{I_m R_m}{I - I_m} \qquad \text{...........(3)}$$

For each required value of full-scale meter current we can then solve for the value of the shunt resistance required.

Example (1) **:** A 1-mA meter movement with an internal resistance of 100 Ω is to be converted into a 0–100 mA ammeter. Calculate the value of the shunt resistance required.

SOLUTION: $I_s = I - I_m = 100 - 1 = 99$ mA

$$R_s = \frac{I_m R_m}{I_s} = \frac{1 \text{ mA} \times 100\ \Omega}{99 \text{ mA}} = 1.01\ \Omega$$

The shunt resistance used with a basic movement may consist of a length of constant-temperature resistance wire within the case of the instrument or it may be an external (manganin or constantan) shunt having a very low resistance. Figure (3) shows an external shunt. It consists of evenly spaced sheets of resistive material welded into a large block of heavy copper on each end of the sheets. The resistance material has a very low temperature coefficient, and a low thermoelectric effect exists between the resistance material and the cop-

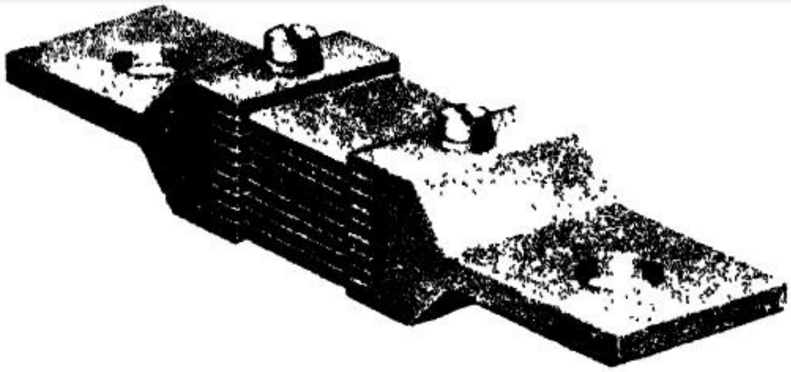


Fig (3)

High current switchboard instrument shunt (*Courtesy Weston Instruments Inc.*).

per. External shunts of this type are normally used for measuring very large currents.

The current range of the dc ammeter may be further extended by a number of shunts, selected by a *range switch*. Such a meter is called a *multirange* ammeter. Figure (4) shows the schematic diagram of a multirange ammeter. The circuit has four shunts, R_a, R_b, R_c, and R_d, which can be placed in parallel with the movement to give four different current ranges. Switch S is a multiposition, ***make-before-break*** type switch, so that the movement will not be damaged, unprotected in the circuit, without a shunt as the range is changed.

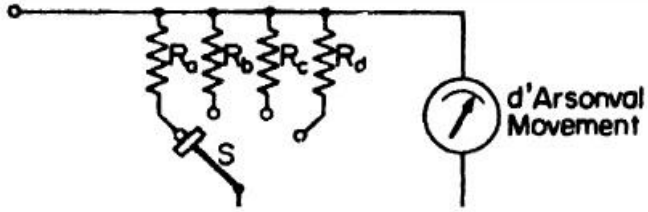


Fig (4)

Schematic diagram of a simple multirange ammeter.

The *universal shunt* or *Ayrton shunt* of Fig. (5) eliminates the possibility of the meter being in the circuit without a shunt. This advantage is gained at the price of a slightly higher over-all meter resistance. The Ayrton shunt provides an excellent opportunity to apply basic network theory to a practical circuit.

Example (2) **:** Design an Ayrton shunt to provide an ammeter with current ranges of 1 A, 5 A, and 10 A. A d'Arsonval movement with an internal resistance $R_m = 50\ \Omega$ and full-scale deflection current of 1 mA is used in the configuration of Fig. (5)

SOLUTION: *On the 1-A range*: $R_a + R_b + R_c$ are in parallel with the 50-Ω movement. Since the movement requires 1 mA for full-scale deflection, the shunt will be required to pass a current of 1 A − 1 mA = 999 mA.

$$R_a + R_b + R_c = \frac{1 \times 50}{999} = 0.05005\ \Omega \qquad \text{(I)}$$

On the 5-A range: $R_a + R_b$ are in parallel with $R_c + R_m$ (50 Ω). In this

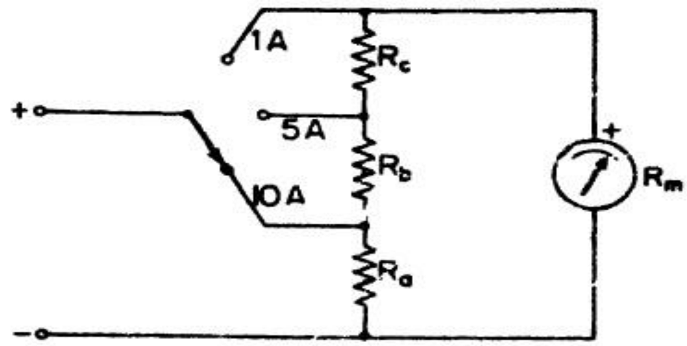


Figure (5)

The universal shunt or Ayrton shunt.

case there will be a 1-mA current through the movement and R_c in series, and 4,999 mA through $R_a + R_b$. Again using

$$R_a + R_b = \frac{1 \times (R_c + 50\ \Omega)}{4,999} \qquad \text{(II)}$$

On the 10-A range: R_a now serves as the shunt and $R_b + R_c$ are in series with the movement. The current through the movement again is 1 mA and the shunt passes the remaining 9,999 mA. Using Eq.

$$R_a = \frac{1 \times (R_b + R_c + 50\ \Omega)}{9{,}999} \qquad \text{(III)}$$

Solving the three simultaneous equations (I, II, and III), we obtain

$$\begin{aligned}
4{,}999 \times \text{(I)}: &\quad 4{,}999\,R_a + 4{,}999\,R_b + 4{,}999\,R_c = 250.2 \\
\text{(II)}: &\quad 4{,}999\,R_a + 4{,}999\,R_b - R_c = 50
\end{aligned}$$

Subtracting (II) from (I):

$$\begin{aligned}
5{,}000\,R_c &= 200.2 \\
R_c &= 0.04004\ \Omega
\end{aligned}$$

Similarly,

$$\begin{aligned}
9{,}999 \times \text{(I)}: &\quad 9{,}999\,R_a + 9{,}999\,R_b + 9{,}999\,R_c = 500.45 \\
\text{(III)}: &\quad 9{,}999\,R_a - R_b - R_c = 50
\end{aligned}$$

Subtracting (III) from (I),

$$10{,}000\,R_b + 10{,}000\,R_c = 450.45$$

Substituting the previously calculated value for R_c into this expression,

$$\begin{aligned}
10{,}000\,R_b &= 450.45 - 400.4 \\
R_b &= 0.005005\ \Omega \\
\text{and} \quad R_a &= 0.005005\ \Omega
\end{aligned}$$

This calculation indicates that for larger currents the value of the shunt resistance may become quite small.

Direct-current ammeters are commercially available in a large number of ranges, from 20 μA to 50 A full-scale for a self-contained meter and to 500 A for a meter with external shunt. Laboratory-type precision ammeters are furnished with a calibration chart, so that the user may correct his readings for any scale errors. Some instruments have accurately drawn, hand-calibrated scales, with the scale marking corresponding to accurately known calibration currents.

Observe the following precautions when using an ammeter in measurement work:

(a) *Never* connect an ammeter *across* a source of emf. Because of its low resistance it would draw damaging high currents and destroy the delicate movement. *Always* connect an ammeter in series with a load capable of limiting the current.

(b) Observe the correct *polarity*. Reverse polarity causes the meter to deflect against the mechanical stop and this may damage the pointer.

(c) When using a multirange meter, first use the highest current range, then decrease the current range until substantial deflection is obtained. To increase accuracy of the observation (see Chapter 1), use the range which will give a reading as near to full-scale as possible.

DC Voltmeters

The addition of a series resistor, or ***multiplier,*** converts the basic d'Arsonval movement into a ***dc voltmeter,*** as shown in Fig. (6) . The multiplier limits the current through the movement so as not to exceed the value of the full-scale-deflection current (I_{fsd}). A dc voltmeter measures the potential difference between two points in a dc circuit and is therefore connected *across* a source of emf or a circuit component. The meter terminals are generally marked "pos" and "neg," since polarity must be observed.

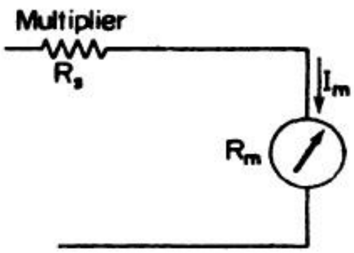


Figure (6)

The basic dc voltmeter circuit.

The value of a multiplier, required to extend the voltage range, is calculated from Fig. (6)

where I_m = deflection current of the movement (I_{fsd})
R_m = internal resistance of the movement
R_s = multiplier resistance
V = full-range voltage of the instrument.

For the circuit of Fig. (6) ,

$$V = I_m(R_s + R_m)$$

Solving for R_s gives

$$R_s = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m \qquad \text{............(4)}$$

The multiplier is usually mounted inside the case of the voltmeter for moderate ranges up to 500 V. For higher voltages, the multiplier may be mounted separately outside the case on a pair of binding posts to avoid excessive heating inside the case.

The addition of a number of multipliers, together with a *range switch*, provides the instrument with a workable number of voltage ranges. Figure (7) shows a *multirange* voltmeter using a four-position switch and four multipliers, R_1, R_2, R_3, and R_4, for the voltage ranges V_1, V_2, V_3, and V_4, respectively. The values of the multipliers can be computed by using the method shown earlier or, alternatively, by the *sensitivity method*. The sensitivity method is illustrated by Example 4 (Sec. 4-7) where sensitivity is discussed.

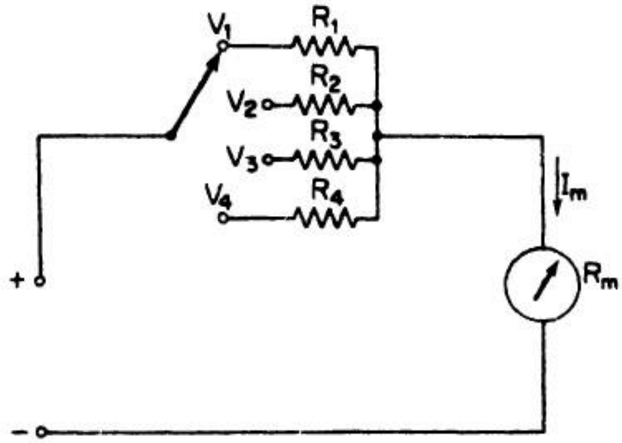


Figure (7)

A multirange voltmeter.

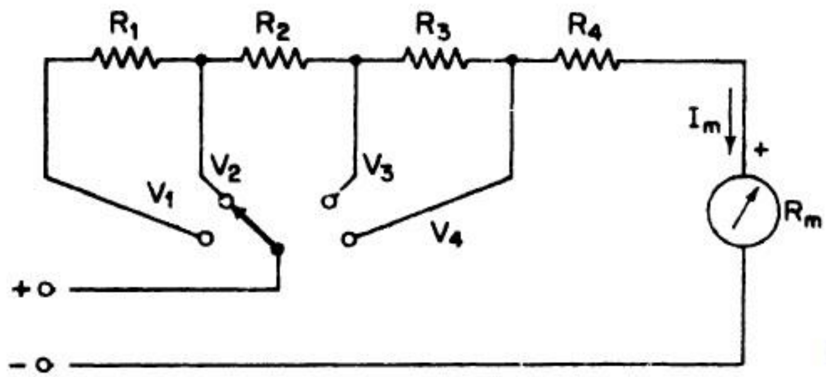


Figure (8)

A more practical arrangement of multiplier resistors in the multirange voltmeter.

A variation of the circuit of Fig. (7) is shown in Fig. (8), where the multipliers are connected in a series string and the range selector switches the appropriate amount of resistance in series with the movement. This system has the advantage that all multipliers except the first have standard resistance values and can be obtained commercially in precision tolerances. The low-range multiplier, R_4, is the only special resistor which must be manufactured to meet the specific circuit requirements:

Example (3) **:** A basic d'Arsonval movement with internal resistance, $R_m = 100\ \Omega$, and full-scale current, $I_{fsd} = 1$ mA, is to be converted into a multirange dc voltmeter with voltage ranges of 0–10 V, 0–50 V, 0–250 V, and 0–500 V.

The circuit arrangement of Fig. (8) is to be used for this voltmeter.

SOLUTION: For the 10-V range (the V_4 position of the range switch), the total circuit resistance is

$$R_T = \frac{10\ V}{1\ mA} = 10\ k\Omega$$

$$R_4 = R_T - R_m = 10\ k\Omega - 100\ \Omega = 9{,}900\ \Omega$$

For the 50-V range (V_3 position of range switch),

$$R_T = \frac{50\ V}{1\ mA} = 50\ k\Omega$$

$$R_3 = R_T - (R_4 + R_m) = 50\ k\Omega - 10\ k\Omega = 40\ k\Omega$$

For the 250-V range (V_2 position of range switch),

$$R_T = \frac{250\ V}{1\ mA} = 250\ k\Omega$$

$$R_2 = R_T - (R_3 + R_4 + R_m) = 250\ k\Omega - 50\ k\Omega = 200\ k\Omega$$

For the 500-V range (V_1 position of range switch),

$$R_T = \frac{500\ V}{1\ mA} = 500\ k\Omega$$

$$R_1 = R_T - (R_2 + R_3 + R_4 + R_m) = 500\ k\Omega - 250\ k\Omega = 250\ k\Omega$$

Voltmeter Sensitivity and Loading Effect

In Sec.DC Voit was shown that the full-scale deflection current I_{fsd} was reached on all voltage ranges when the corresponding full-scale voltage was applied. As shown in Example 4-4, a current of 1 mA is obtained for voltages of 10 V, 50 V, 250 V, and 500 V across the meter terminals. For each voltage range, the quotient of the total circuit resistance R_T and the range voltage V is always 1,000 Ω/V. This figure is often referred to as the ***sensitivity*** or the ***ohms-per-volt rating*** of the voltmeter. Note that the sensitivity, S, is essentially the ***reciprocal*** of the full-scale deflection current of the basic movement, or

$$S = \frac{1}{I_{fsd}} \frac{\Omega}{\mathrm{V}} \qquad \text{...............(5)}$$

The sensitivity S of the voltmeter may be used to advantage in the ***sensitivity method*** of calculating the resistance of the multiplier in a dc voltmeter. Consider the circuit of Fig. (8) ,

where S = sensitivity of the voltmeter, in Ω/V
V = the voltage range, as set by the range switch
R_m = internal resistance of the movement (plus the previous series resistors)
R_s = resistance of the multiplier.

For the circuit of Fig. (8) ,

$$R_T = S \times V$$

and

$$R_s = (S \times V) - R_m \qquad \text{.............(6)}$$

Use of the sensitivity method is illustrated in Example 4-5.

Example (4) : Repeat Example (3), now using the sensitivity method for calculating the multiplier resistances.

SOLUTION:

$$S = \frac{1}{I_{fsd}} = \frac{1}{0.001\ \text{A}} = 1{,}000\,\frac{\Omega}{V}$$

$$R_4 = (S \times V) - R_m = \frac{1{,}000\ \Omega}{\text{V}} \times 10\ \text{V} - 100\ \Omega = 9{,}900\ \Omega$$

$$R_3 = (S \times V) - R_m = \frac{1{,}000\ \Omega}{\text{V}} \times 50\ \text{V} - 10{,}000\ \Omega = 40\ \text{k}\Omega$$

$$R_2 = (S \times V) - R_m = \frac{1{,}000\ \Omega}{V} \times 250\ V - 50\ k\Omega = 200\ k\Omega$$

$$R_1 = (S \times V) - R_m = \frac{1{,}000\ \Omega}{V} \times 500\ V - 250\ k\Omega = 250\ k\Omega$$

The sensitivity of a dc voltmeter is an important factor when selecting a meter for a certain voltage measurement. A low-sensitivity meter may give correct readings when measuring voltages in low-resistance circuits but is certain to produce very unreliable readings in high-resistance circuits. A voltmeter, when connected across two points in a highly resistive circuit, acts as a shunt for that portion of the circuit and thus reduces the equivalent resistance in that portion of the circuit. The meter will then give a lower indication of the voltage drop than actually existed before the meter was connected. This effect is called the *loading effect* of an instrument and is caused principally by *low-sensitivity* instruments. The loading effect of a voltmeter is illustrated in Example (5) .

Example (5) **:** It is desired to measure the voltage across the 50-kΩ resistor in the circuit of Fig. (9) . Two voltmeters are available for this

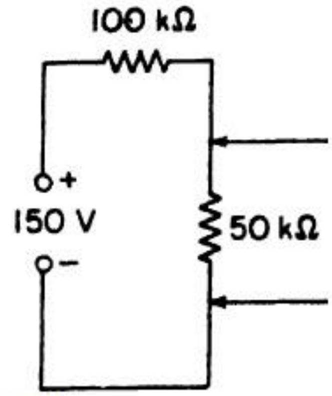


Figure (9)

Voltmeter loading effect.

measurement: voltmeter 1 has a sensitivity of 1,000 Ω/V and voltmeter 2, 20,000 Ω/V. Both meters are used on their 50-V range. Calculate (a) the reading of each meter; (b) the error in each reading, expressed as a percentage of the true value.

SOLUTION: Inspection of the circuit indicates that the voltage across the 50-kΩ resistor is

$$\frac{50\ \mathrm{k\Omega}}{150\ \mathrm{k\Omega}} \times 150\ \mathrm{V} = 50\ \mathrm{V}$$

This is the ***true*** value of voltage across the 50-kΩ resistor.

(a) *Voltmeter 1* ($S = 1{,}000\ \Omega/\mathrm{V}$) has a resistance of $50\ \mathrm{V} \times 1{,}000\ \Omega/\mathrm{V} = 50\ \mathrm{k\Omega}$ on its 50-V range. Connecting the meter across the 50-kΩ resistor causes the equivalent parallel resistance to be decreased to 25 kΩ and the total circuit resistance to 125 kΩ. The potential difference across the combination of meter and 50-kΩ resistor is

$$V_1 = \frac{25\ \text{k}\Omega}{125\ \text{k}\Omega} \times 150\ \text{V} = 30\ \text{V}$$

Hence, the voltmeter indicates a voltage of 30 V. *Voltmeter 2* ($S = 20$ kΩ/V) has a resistance of 50 V × 20 kΩ/V = 1 mΩ on its 50-V range. When this meter is connected across the 50-kΩ resistor, the equivalent parallel resistance equals 47.6 kΩ. This combination produces a voltage of

$$V_2 = \frac{47.6\ \text{k}\Omega}{147.6\ \text{k}\Omega} \times 150\ \text{V} = 48.36\ \text{V}$$

which is indicated on the voltmeter.

(b) *Voltmeter 1* indicates an error of

$$\%\ \text{error} = \frac{\text{true voltage} - \text{apparent voltage}}{\text{true voltage}} \times 100\%$$

$$= \frac{50\ \text{V} - 30\ \text{V}}{50\ \text{V}} \times 100\% = 40\%$$

Voltmeter 2 indicates an error of

$$\%\ \text{error} = \frac{50\ \text{V} - 48.36\ \text{V}}{50\ \text{V}} \times 100\% = 3.28\%$$

The computation of Example (5) indicates that the meter with the higher sensitivity or ohms-per-volt rating gives the most *reliable* result. It is important to realize the factor of sensitivity, particularly in cases where voltage measurements are made in high-resistance circuits.

The matter of reliability and accuracy of the test result raises an interesting point. When an *insensitive, yet highly accurate,* dc voltmeter is placed across the terminals of a high resistance, the meter accurately reflects the voltage condition produced by loading. The error is a *human error* or gross error because the proper instrument was not selected. The meter "disturbs" the circuit, and the ideal of instrumentation, at all times, is to measure a condition without affecting it in any way. But the human investigator has the responsibility to select an instrument which is precise, reliable, and sufficiently sensitive not to disturb that which is being measured. The fault lies not with the highly accurate instrument but with the investigator, who is using it incorrectly. In fact, the sophisticated instrument user could calculate the true voltage using an insensitive yet accurate meter.

Therefore, *accuracy* is always required in instruments; *sensitivity* is needed only in special applications where loading disturbs that which is being measured. Example (6) illustrates how an insensitive, yet accurate instrument is used to perform an entirely valid measurement.

Example (6) **:** The only voltmeter available in a laboratory has a sensitivity of 100 Ω/V and three scales, 50 V, 150 V, and 300 V. When connected in the circuit of Fig. (10), the meter reads 4.65 V on its lowest (50-V) scale. Calculate the value of R_x.

SOLUTION: The equivalent resistance of the voltmeter on its 50-V scale is

$$R_V = 100\,\frac{\Omega}{V} \times 50\text{ V} = 5\text{ k}\Omega$$

Let R_p = the parallel resistance of R_x and R_V.

$$R_p = \frac{V_p}{V_s} \times R_s = \frac{4.65}{95.35} \times 100\text{ k}\Omega = 4.878\text{ k}\Omega$$

Then,

$$R_x = \frac{R_p \times R_v}{R_v - R_p} = \frac{4.878\ \text{k}\Omega \times 5\ \text{k}\Omega}{0.122\ \text{k}\Omega} = 200\ \text{k}\Omega$$

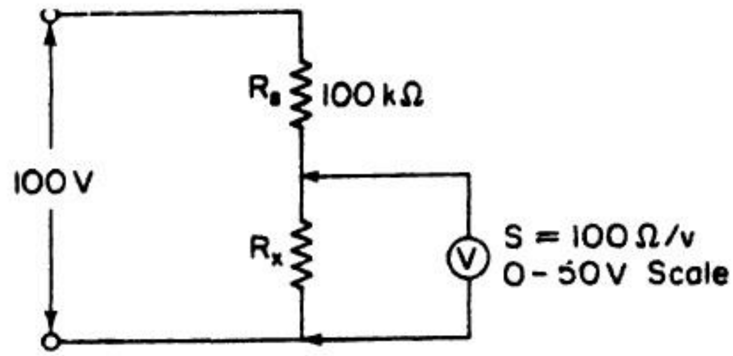


Figure (10)

The use of an accurate but insensitive voltmeter to determine the resistance of R_x.

Chapter Three

I1 = (7.6 – 0.7)/3300 = 2 mA
Vp to p = I1 * T/C = 2*10^-3 * 10^-3/(0.5*10^-6) = 4 V
Q4: by doubling the value of the capacitor. By using variable resistance instead of the constant resistor R3. The value of the variable resistance is equal to
3300-330 ≤ R3 ≤ 3300+330 , since 330 = 10% of 3300.
-Measurement of Voltage, Frequency and Phase:

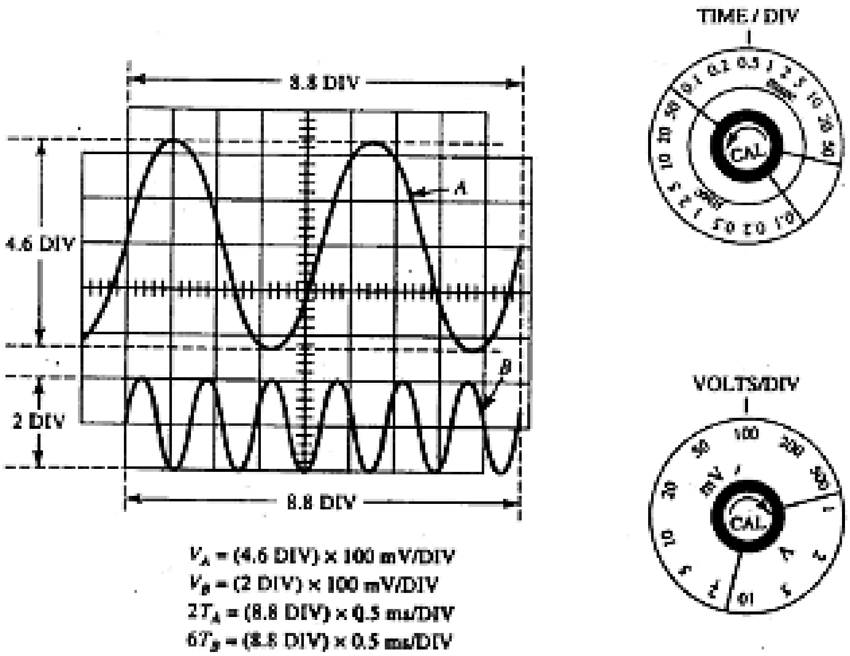


Fig.6-11 The peak to peak voltage of a waveform is measured by multiplying the VOLT/DIV setting by the peak to peak vertical division occupied by the waveform. The time period is determined by multiplying the horizontal division for one cycle by the TIME/DIV.
Wave A T= 4.4 divisions * 0.5 ms = 2.2 ms
Frequency f = 1/ 2.2 ms =455 Hz
Wave B T= 1.46 divisions * 0.5 ms = 0.73 ms
f = 1/0.73 = 1.36 kHz.

Phase Measurement: The phase difference between two waveforms is measured by the method illustrated by Fig.6-12. Each period has a time period of 8 horizontal divisions, and the time between commencements of each cycle is 1.4

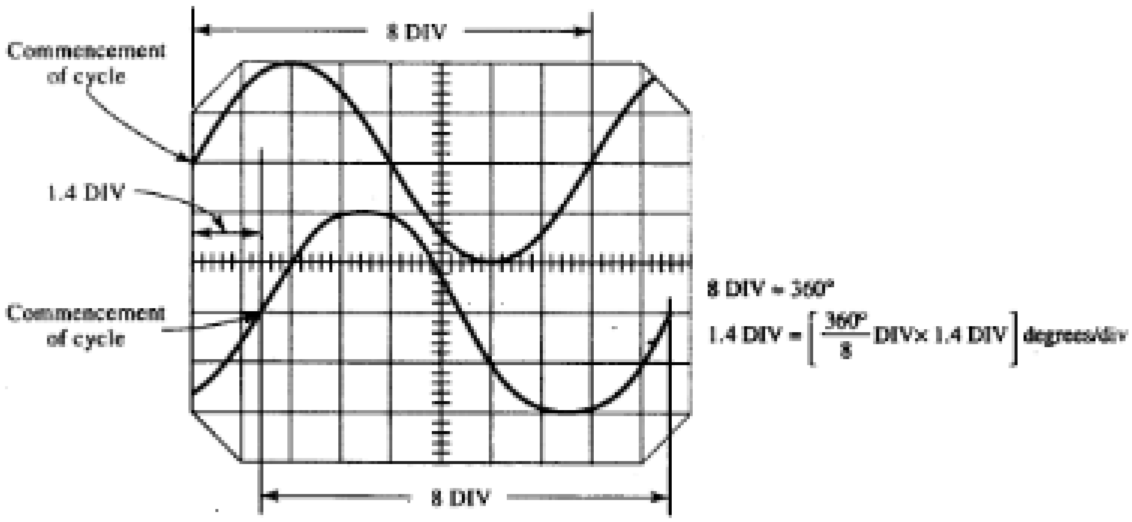
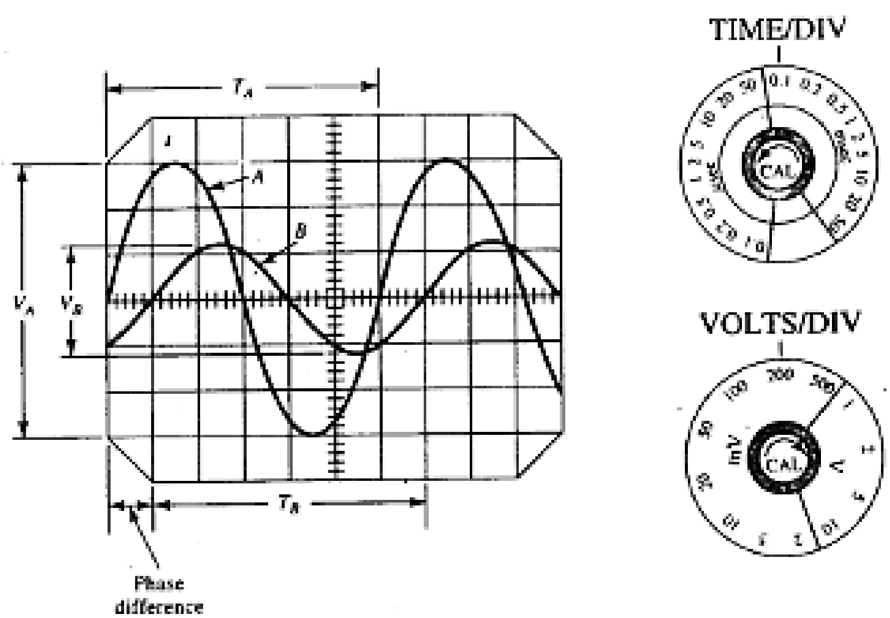
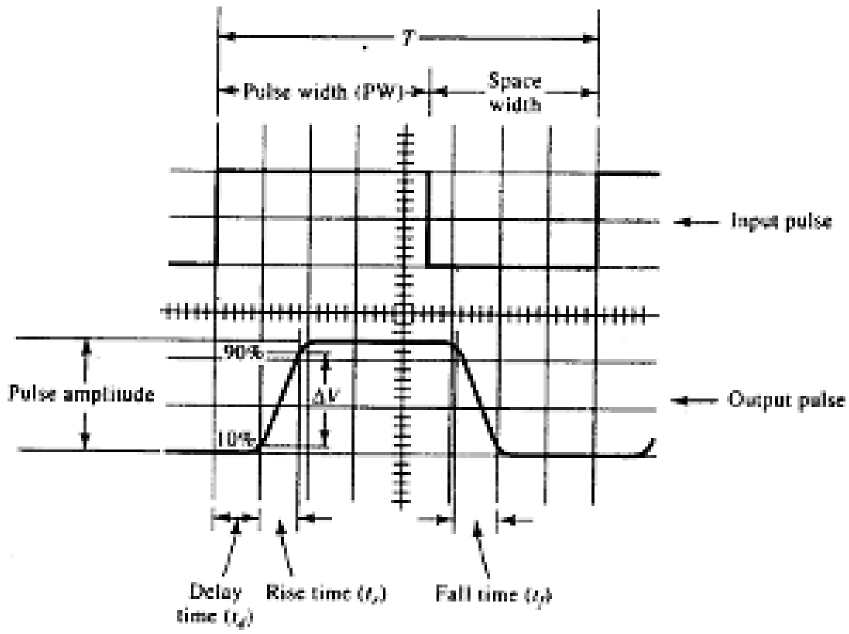


Fig.6-12 The phase difference between two sine waves divisions. One cycle = 360º. Therefore, 8 div. = 360º,
1div. = 360/8 = 45º
Thus the phase difference is
ϴ = 1.4 div * 45º =63º
So ϴ = phase difference in division * degree/div.
Example: Determine the amplitude, frequency and phase difference between the two waveforms illustrated below.

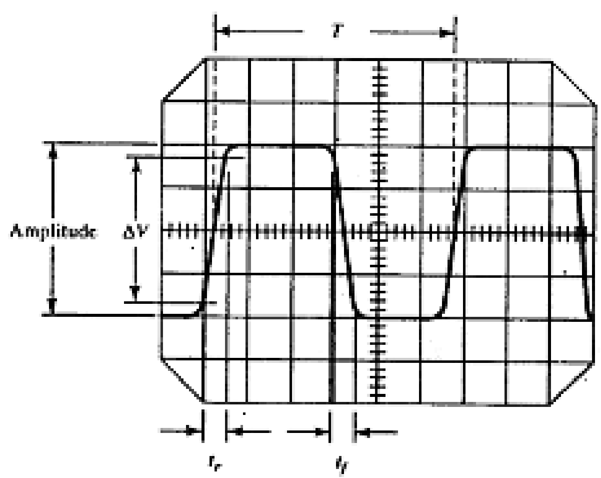


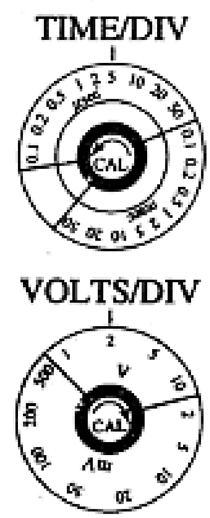
Pulse Measurements

- Pulse Amplitude, Pulse Width and Space Width:



- Rise Time, fall time and Delay Time:





Chapter seven

Signal Generator

Signal generators are widely used to provide test signals to check the performance of electronic systems.

Signal source that generates only sinusoidal waveform is called oscillator. The signal source that not only generates sinusoidal waveforms but also square, pulse and triangular is called function generator.

- Principle of Oscillators:

The block diagram of an oscillator is shown in Fig.7-1. At certain frequency a positive feedback occurs and there is no need for Vin. It can be shown that the gain equation for the + ve feedback circuit is given by:

$$Af.b = \frac{A}{1-\beta A} = gain\,with\,feedback \quad \text{---1}$$

Where A= open loop gain and β = feedback factor = Vin / Vo

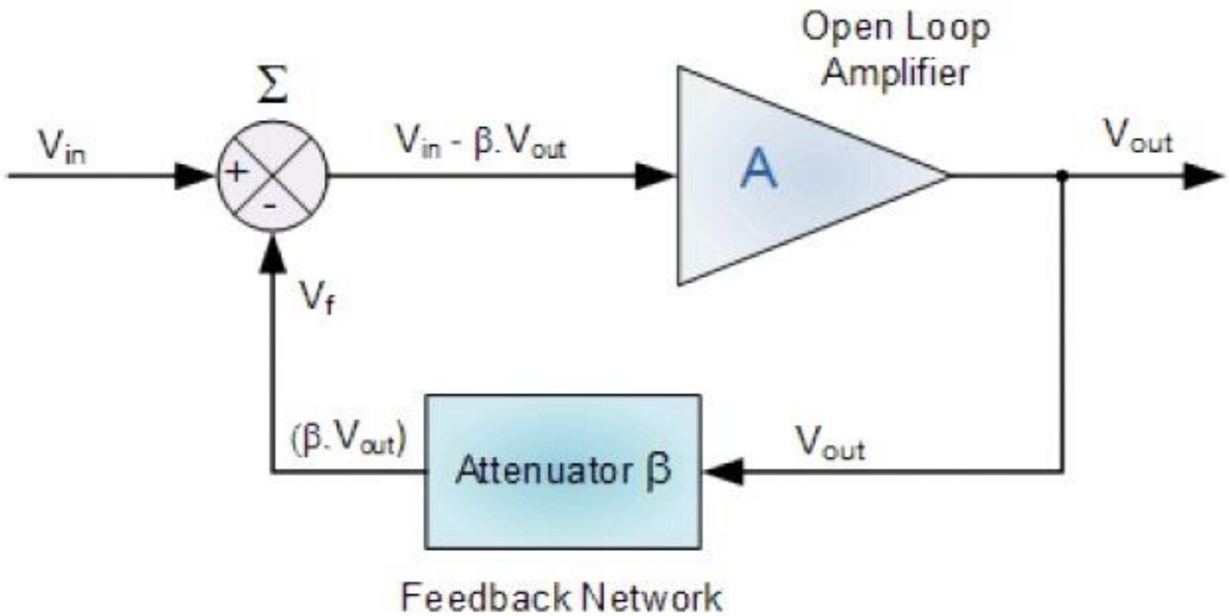


Fig.7-1 Oscillator Block Diagram

The Barkhausen criteria requirements for oscillation are: 1.The net phase shift in the circuit is zero 2. $\beta A \geq 1$, or $A \geq 1/\beta$.

Example1: The amplifier of Fig.7-1 has the following value $|A|=20$, $|\beta|=0.05$. Determine the closed loop gain of the amplifier.

sol. using Eq.1 $Af.b = A/(1-\beta A) = 20/(1-(0.05*20)) = \infty$

Types of oscillators:

There are two types of oscillators: 1. High frequency oscillator which uses LC and 2. Low frequency oscillator which uses RC as illustrated below:

LC- Oscillators, RF high frequency (from 10kHz to 500MHz). 1.Hartly 2. Colpitts oscillator.
RC- Oscillators, audio low frequency (up to 10MHz). 1. Wien Bridge 2. Phase-shift oscillator.

- Radio Frequency Oscillators: The LC tank circuit forms the basic LC-oscillator. At the resonance frequency (frequency of oscillation), the tank circuit appears resistive; i.e, Xc=XL. Two types of RF oscillators are considered here, Hartly and Colpitts oscillators.
-Hartly Oscillator: Fig.7-2 shows the circuit of Hartly oscillator. For a Hartly oscillator to start up and oscillate at proper frequency, the voltage gain (A) of the circuit must

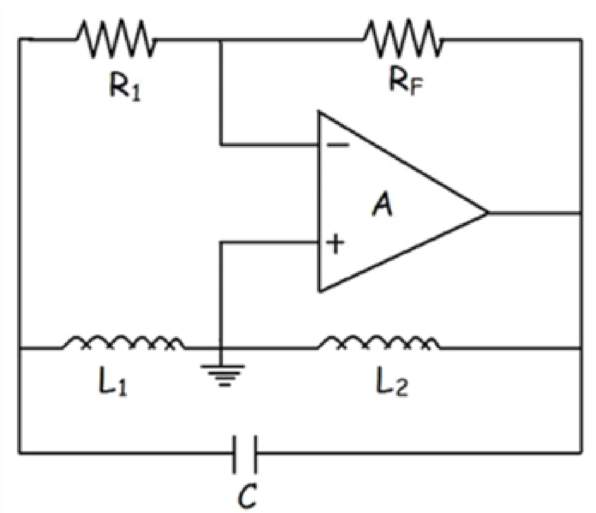


Fig.7-2 Hartly oscillator
meet the Barkhausen criteria.
$A\beta \geq 1$ so $A \geq 1/\beta$ and the gain $A = Rf/R1$
$L = L1 + L2 + 2M$
$Vo = VL2 = I\ XL2$
$\beta = Vi / Vo = XL1 / XL2 = wL1 / wL2 = L1/L2$ so $1/\beta = L2/L1$
At resonance frequency XL = Xc so $2\pi\, fo\, L = 1/(2\pi fo\, C)$
$fo = 1/(2\pi\sqrt{LC})$

- Colpitts Oscillator: The circuit of Colpitts oscillator is shown in Fig.7-3. To start up and oscillate at the resonance frequency the Barkhausen criteria must be satisfied, i.e,
$A\beta \geq 1$ or $A \geq 1/\beta$, $1/\beta = C1/C2$ so $A = Rf/R1 \geq C1/C2$

$V_o = V_{C2} = I\,X_{C2}$ and $V_i = V_{C1} = I\,X_{C1}$
$\beta = V_i / V_o = X_{C1} / X_{C2} = \frac{1/wC1}{1/wC2} = C2/C1$

$1/\beta = C1/C2$ at resonance frequency XL = XC
$2\pi f_o L = 1/(2\pi f_o C)$ so $f_o = \frac{1}{2\pi\sqrt{LC}}$
where C = C1* C2/(C1 +C2)

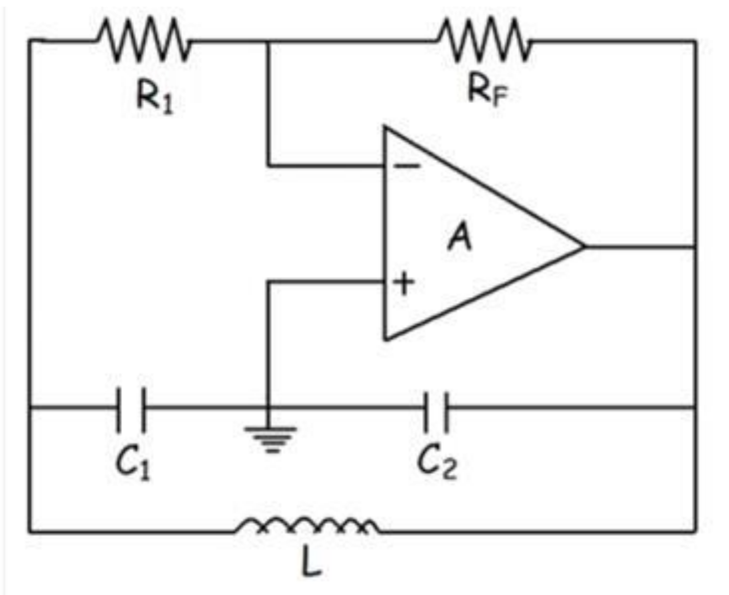


Fig.7-3 Colpits Oscillator
Example2: Design a Colpits oscillator such that the oscillator frequency is equal to 100kHz.
Sol. $f_o = 1/(2\pi\sqrt{LC})$
chose L = 100 µH, then calculate C and if C1=C2 then C1=C2=C/2 . The gain A = R2/R1 should be ≥ 1. Assume that A = 2, chose R1=10kΩ so R2=20KΩ.
Example3: Design a Hartly oscillator such that the oscillator frequency is equal to 500kHz. Assume the mutual inductance between the coils equal zero.
Audio frequency oscillator:

- Wien Bridge Oscillator:
The circuit of the Wien bridge is shown in Fig.7-4. At certain frequency fo the reactive branches produce lead and lag voltage cancellation.
A= 1+ R1/R2 ----------------------2
then

β(jw) = Vf/ Vo =($\frac{R\left(\frac{1}{jwc}\right)}{R+\frac{1}{hwc}})/(\left(R+\frac{1}{jwc}\right)+\left(\frac{\frac{R}{jwc}}{R+\frac{1}{jwc}}\right)) = R/(3R+j(wcR^2-\frac{1}{wc})$ **---------3**

From the Barkhausen principle, we know that A(jw)β(jw) should be a real number and the phase shift must be zero. From equation (2), A is real number, therefore the imaginary part of equation (3) must equal to zero. From which

$$wo\ cR^2-\frac{1}{wc}=0 \quad , \quad wo=\frac{1}{Rc} \quad , \text{so } fo=\frac{1}{2\pi RC}$$

This is equal to the oscillation frequency.
Now from equation (2) and at oscillation frequency, β=1/3 , then if we consider the barkhausen criteria which state that oscillation will happen when |
Therefore the gain of the Wien Bridge is

$$A=\left(1+\frac{R1}{R2}\right)\geq 3 \ ,$$

$$R1/R2\geq 2$$

From the above mentioned, in order to let the Wien Bridge oscillator to oscillate, the gain A must be greater than 3 and the frequency of oscillation is

$$fo = 1/(2\pi Rc)$$

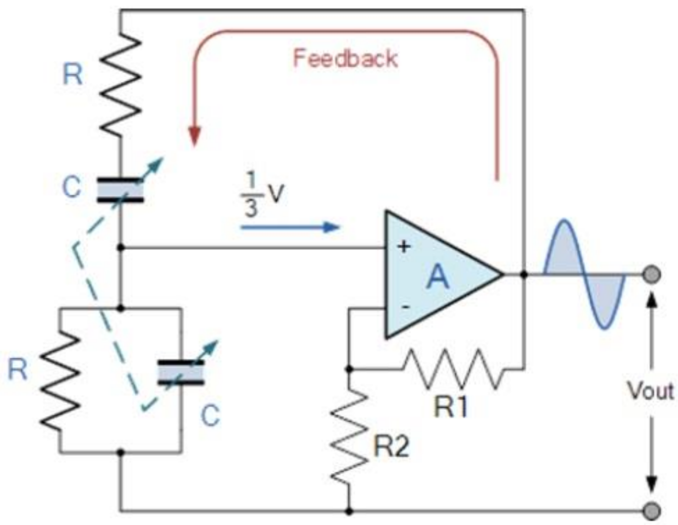


Fig.7-4 Wien Bridge Oscillator

Example4: Design a Wien bridge oscillator with a resonance frequency of 4 kHz.

Sol. R1/R2 ≥2, chose R1=10kΩ so R2=4.7kΩ

4000 = 1/(2лRC) , chose C=0.01µf, so R =3.9kΩ

Example5: If the frequency of the Wien bridge oscillator in Ex.1 should change from 2 to 4kHz. What range of values should be used for the two ganged variable capacitor?

- Phase shift Oscillator:

A phase shift oscillator is shown in Fig.7-5. The amplifier produces 180º phase shift and the feedback at certain frequency will produce an additional phase shift of 180º. Then the net phase shift is zero. The RC- feedback network produces a losses of 1/29, so the amplifier must produce a gain (A) greater than 29 (A≥29) to overcome the RC losses. It can be shown that

Chapter four

Analog Electronic Instruments

- It uses the basic PMMC movement

- It uses an amplifier as an input before the movement to increase the input impedance of the instrument, so there is no loading effect. Also the electronic

instrument can measure low level signal, thus increase the sensitivity of the meter.

- A block diagram of a meter is shown in fig.4-1. The input voltage is amplified and applied to the meter. If the amplifier has a gain of 10, the sensitivity of the measurement is increased by 10 also. If the gain is increased, this meter can be used to measure very small currents and voltages such as microvolts or Nano- amperes.

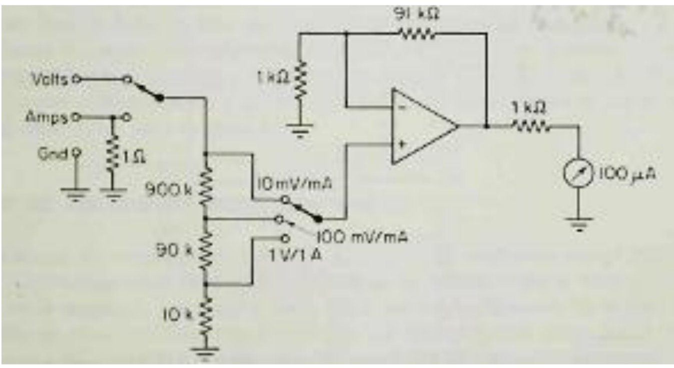
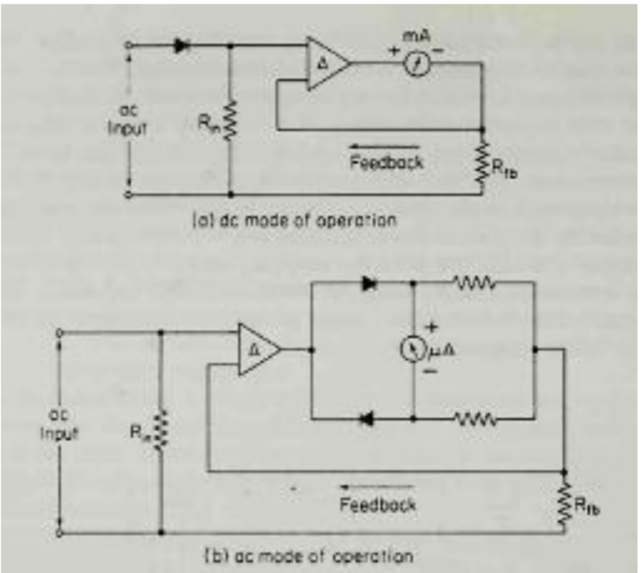


Fig.4-1 Amplified voltage and current meter

-Electronic AC voltmeter using rectifier:

Same as dc voltmeter except that the ac input voltage must be rectified before it can be applied to the dc meter. Rectifier can be taken place before amplification, as shown in fig.4-2a or the ac signal is rectified after amplification, as shown in fig.4-2b.



- Fig.4-2 Basic ac voltmeter circuit

Ac voltmeters are usually of the average responding type, with the meter scale calibrated in terms of the rms value of a sine wave. Non-sinusoidal waveform, however, will cause this type of meter to read high or low depending on the form factor of the waveform.

A few basic rectifier circuits are shown in Fig.4-3

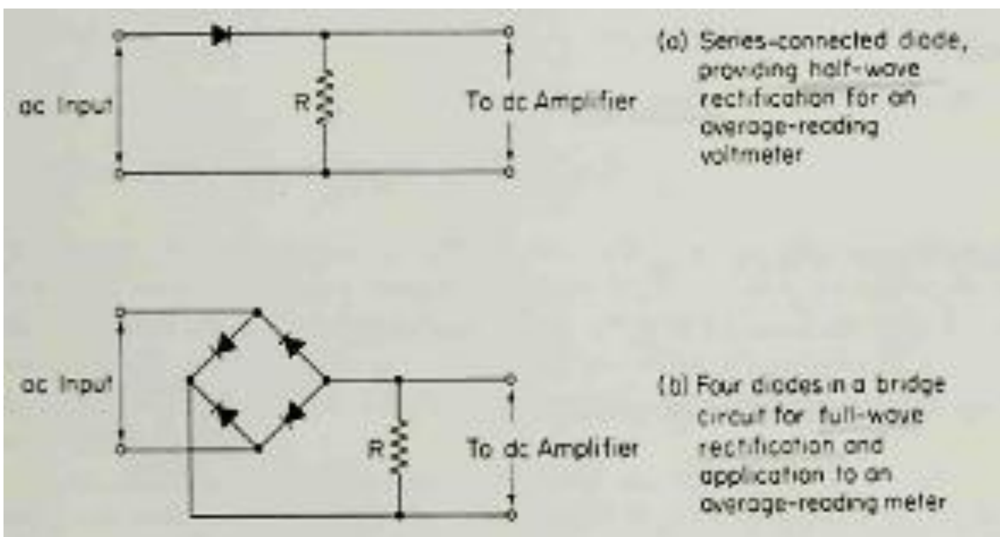
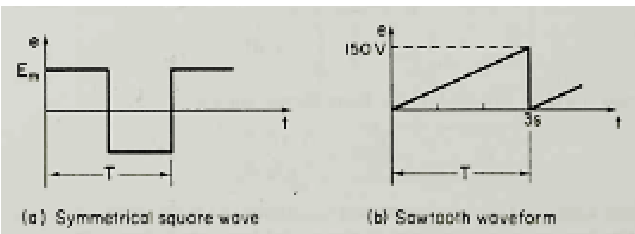


Figure 4-3 Rectifier circuits used in ac voltmeter

Q: The symmetric square-wave voltage of fig.4-4a is applied to an average responding ac meter calibrated in terms of rms value of a sine wave. Calculate a. the form factor of the square-wave voltage b. the error in the meter indication.



Q2: Repeat Q1 if the voltage applied to the meter consists of a saw tooth waveform with peak value of 150V and a period of 3s as shown in fig.4-4b.

Q3: what is the rms of a pulse waveform of 5v peak and a 25 per cent duty cycle.

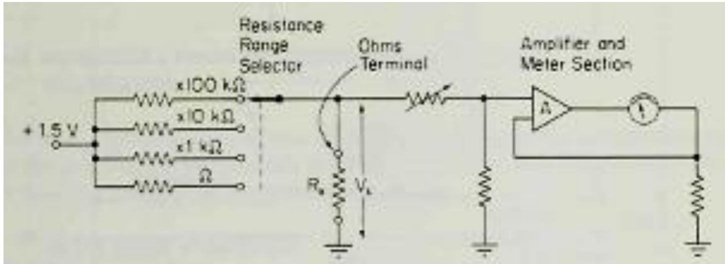
[see cooper]

Electronic Ohmmeter: A typical circuit of an electronic ohmmeter is shown in Fig.4-5, where a separate divider network provides for a number of different resistances ranges, when the unknown resistance Rx is connected to the ohmmeter terminals, the 1.5V battery

supplies current through one of the range resistors and the unknown resistor to ground. Voltage drop Vx across Rx is applied to the input of the amplifier

and causes a deflection on the meter. Since the voltage drop across Rx is directly proportional to its resistance, the meter scale can be calibrated in terms of resistance.

Fig.4-5 an electronic Ohmmeter



Questions:

Q1: What are the advantages of electronic instrumentation devices?

Q2: What is the lowest full-scale voltage that could be displayed with a 100µA meter movement with an internal resistance of 150? What would the sensitivity of this meter be in ohms per volt?[ans. 15mv , 10000Ω/v]

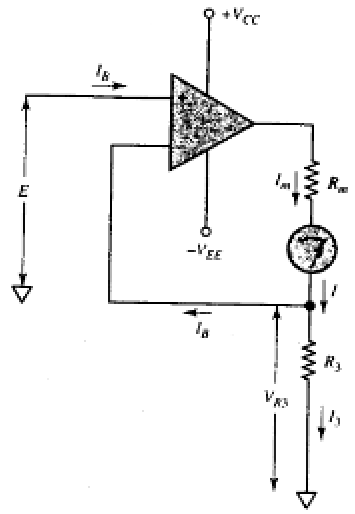
Q3: A 2-mA full-scale current PMMC meter with an internal resistance of 100Ω is available for constructing an ac voltmeter of 200v rms using four diodes in bridge arrangement, where each diode has a forward resistance of 500Ω and infinite reverse resistance, calculate the necessary series-limiting resistance for the 200v rms range.

Solution:[see fig.4-3b]

Vdc = 0.9*200 = 180v (since PMMC is responding to the average value)

( )

Q4: Calculate the value of R3 for the circuit shown below if E=1v is to give FSD on the meter. The moving coil has IFSD=1mA and Rm=100Ω. Also determine the maximum voltage at the op-Amp output terminal.



Solution Q4: R3 =E/IFSD =1/1ma =1kΩ

Vo = I (R3 + Rm) = 1ma(1000 +100) =1.1 volt

Or Vo = Vi (1 + Rm/R3) = 1.1 volt.

-Digital Voltmeter

Digital instrument give readings in the form of digits, i.e in the form of a number, instead of pointer deflections scale as in analog devices. The advantages of these instruments are a reduction in the human reading error, faster reading, better resolution and increase accuracy.

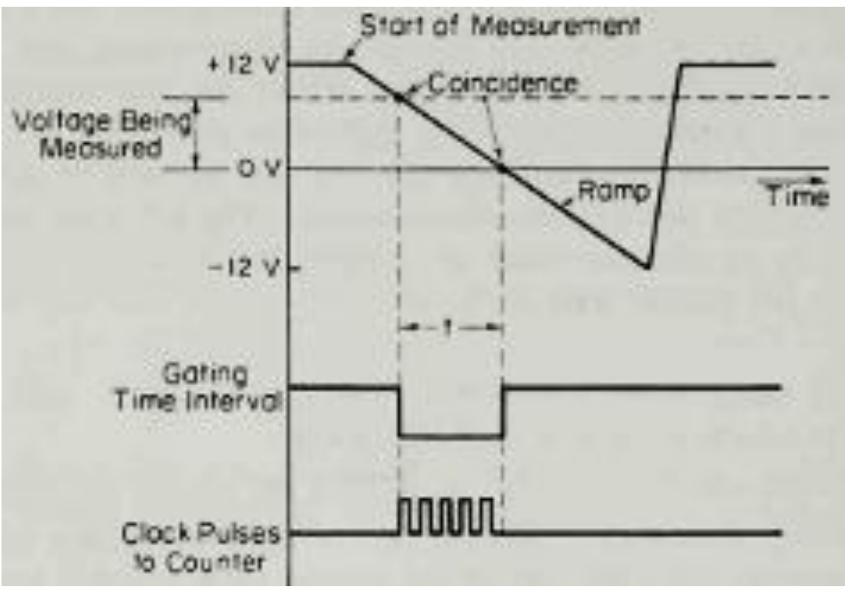
- Ramp-type DVM: the operating principle is based on the measurement of the time it takes for a linear ramp voltage to rise from 0v to the level of the input voltage, or to decrease from the level of the input voltage to zero. This time is measured with an electronic time interval counter, and the count is displayed as a number of digits on electronic indicating display.

Conversion from a voltage to a time is illustrated by the waveform diagram of fig.4-6. At the start of the measurement cycle, a ramp voltage is initiated. The negative-going ramp shown in fig.4-6, is continuously compared with the unknown input voltage. At the instant that the ramp voltage equals the unknown voltage, a comparator circuit generate a pulse which open the gate.

This gate is shown in block diagram of fig.4-7. The ramp voltage continues to decrease with time until it finally reaches 0v or ground and a second comparator generates an output pulse which closes the gate.

An oscillator produces clock pulses which are allowed to pass through the gate to a number of decade counting units which count the number of pulses passed through the gate. The decimal number, displayed by the indicator is a measure of the magnitude of the input voltage.

Fig.4-6 Voltage to time conversion



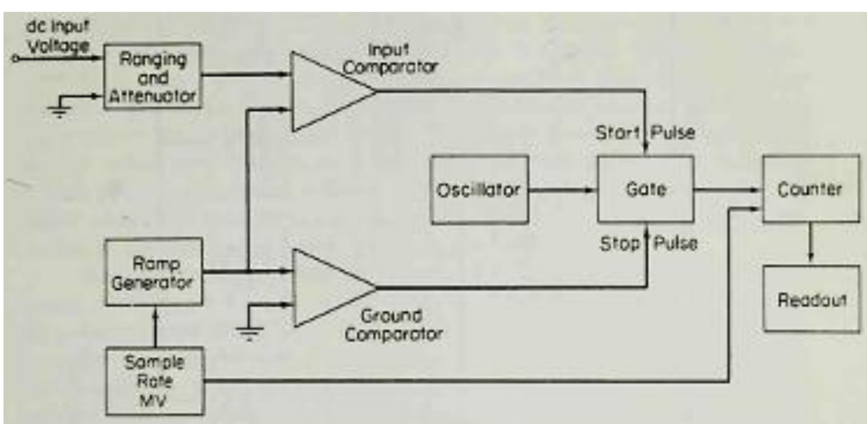
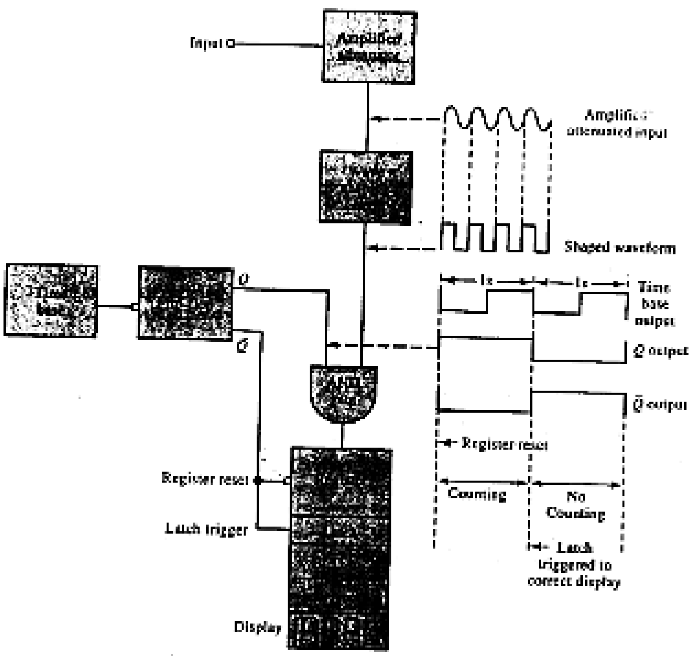


Fig.4-7 Block diagram of a ramp-type DVM

-Digital frequency meter:

If a pulse waveform (clock) is used to trigger a digital counter for a time period of exactly 1 second, the counter registers the number of pulses that occur per second; that is, it registers the frequency of the input waveform. If the count is 1000, the frequency is 1000 pulses per second (1000Hz). The block diagram of a digital frequency counter is shown in fig.4-8 below.



(a) Frequency meter system (b) System waveforms

Chapter five

Resistance measurements and Bridges

1. Ammeter-Voltmeter method:

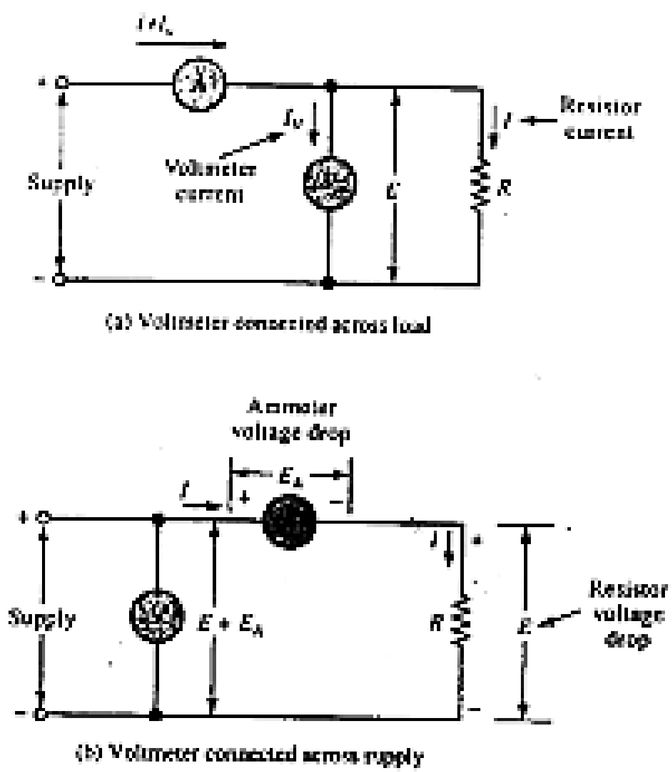


Fig.5-1 Ammeter-Voltmeter method of measuring resistance.

When the voltmeter is connected across the load (a), the ammeter measures the voltmeter and resistor current. When the voltmeter connected across the supply (b), the voltmeter measures the ammeter voltage drop as well as resistor voltage.

Example1:

A resistance is measured by the ammeter and voltmeter circuit illustrated in Figure 7-1(b). The measured current is 0.5 A, and the voltmeter indication is 500 V. The ammeter has a resistance of $R_a = 10\ \Omega$, and the voltmeter on a 1000 V range has a sensitivity of 10 kΩ/V. Calculate the value of R.

Solution

$$E + E_A = 500\ \text{V}$$

$$I = 0.5\ \text{A}$$

$$R_a + R = \frac{E + E_A}{I}$$

$$= \frac{500\ \text{V}}{0.5\ \text{A}} = 1000\ \Omega$$

$$R = 1000\ \Omega - R_a$$

$$= 1000\ \Omega - 10\ \Omega$$

$$= 990\ \Omega$$

Example2:

If the ammeter, voltmeter, and resistance R in Example 7-1 are reconnected in the form of Figure 7-1(a), determine the ammeter and voltmeter indications.

Solution

$$R_v = 1000\ \text{V} \times 10\ \text{k}\Omega/\text{V}$$
$$= 10\ \text{M}\Omega$$

$$R_v \| R = 10\ \text{M}\Omega \| 990\ \Omega$$
$$= 989.9\ \Omega$$

$$\textit{supply voltage} = 500\ \text{V}$$

$$\textit{voltmeter reading} = E = \frac{500\ \text{V} \times R_v \| R}{R_a + R_v \| R}$$
$$= \frac{500\ \text{V} \times 989.9\ \Omega}{10\ \Omega + 989.9\ \Omega}$$
$$= 495\ \text{V}$$

$$\textit{ammeter reading} = I + I_v = \frac{E}{R_v \| R}$$
$$= \frac{495\ \text{V}}{989.9\ \Omega}$$
$$\approx 0.5\ \text{A}$$

Bridges and their applications:

Bridges circuits are used for measuring component values, such as resistance, inductance and capacitors. It can be used to measure frequency and phase angle.

-Wheatstone Bridge: the circuit diagram of the Wheatstone bridge is shown in Fig.5-2.

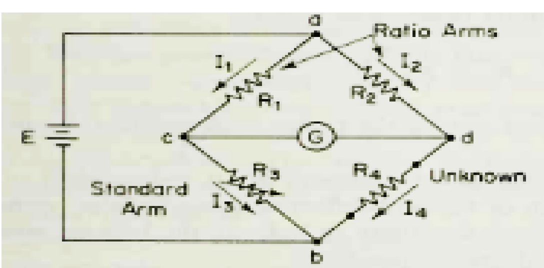


Fig.5-2 Schematic diagram of the Wheatstone bridge circuit.

The bridge is said to be balance when the potential difference across the galvanometer (G) is 0 V so that there is no current through the G. Hence the bridge is balanced when

I1 * R1 = I2 * R2

Also

I1 = I3 =

Combining the above equations:

R1 /(R1+R3) = R2/(R2+R4)

From which

R1 * R4 = R2 * R3 and if R4 = Rx is the unknown resistance, its resistance Rx can be expressed

Rx = R3 * R2/ R1

Resistor R3 is called the standard arm of the bridge, and resistor R1 and R2 are called the ratio arms.

-Source of measurement error in Wheatstone bridge:

1. Errors of the three known resistors.

2. Insufficient sensitivity of the null detector (G).

3. Changes in resistances of the bridge due to heating effect of the current through the resistors.

4. Thermal emf in the bridge circuit and the (G) can cause problems when low value resistors are being measured.

5. Errors due to the resistance leads and contacts.

-The sensitivity of the Whetstone Bridge can be defined as the smallest change in the unknown resistor that can be detected or read by the galvanometer of the bridge.

-Sensitivity of the whetstone bridge depends on the current sensitivity of the galvanometer (mm/µA), the galvanometer resistance and the bridge supply voltage.

-Example: Fig.5-3 shows schematic diagram of Whetstone bridge with its values. The G has a current sensitivity of 10mm/µA and an internal resistor of 100Ω. Calculate the deflection of G caused by the 5Ω unbalance in the arm BC.

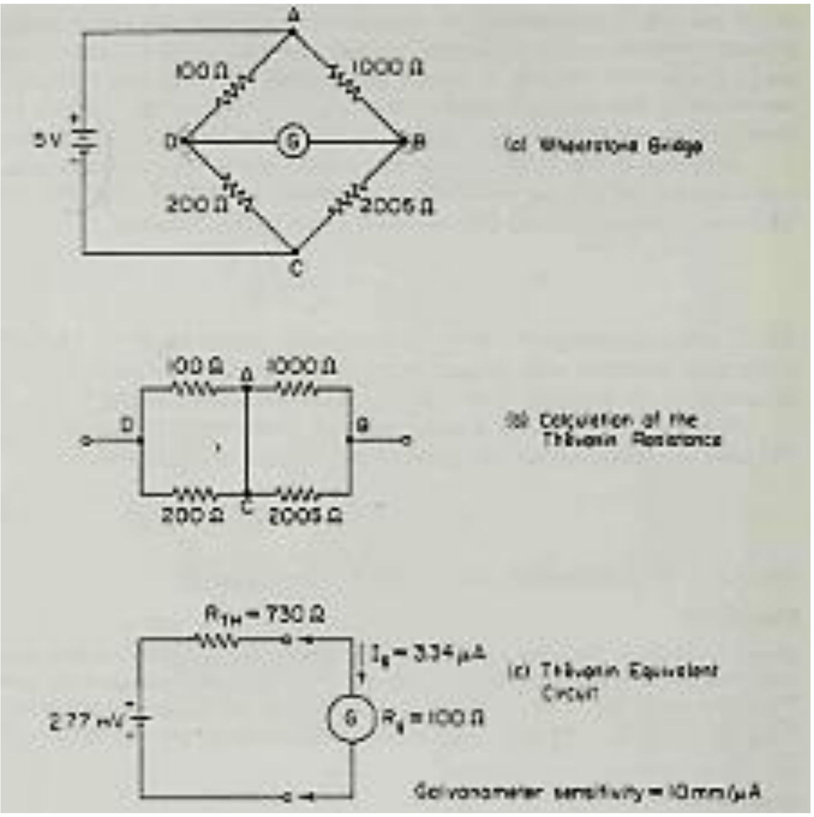


Fig.5-3 Application of Thevenin's theorem to the bridge (a) Whetstone bridge (b)

Thevenin resistor looking into terminals c and d. (c) Complete Thevenin circuit, with the G connected to terminals c and d.

Rth = 100//200 + 1000//2005 = 730Ω

= 3.34 µA Ig = Eth /(Rth + Rg) =

The G deflection (D) is

D= 3.34µA * 10mm/µA=33.4mm

Example: The G of the above example is replaced by one with internal resistance of 500Ω and a current sensitivity of 1mm/µA. assuming that a deflection of 1mm

can be observed on the G scale determine if this new G is capable of detecting the 5Ω unbalance in arm BC of Fig.5-3(a).

Sol. Ig = Eth /(Rth +Rg) = = 2.25 µA

The G deflection (D) is 2.25 µA*1mm/µA =2.25 mm.

This deflection can be easily observed.

Problems:

Q1: Determine the value of the unknown resistor Rx in Fig.5-2 if R1 =16kΩ, R2 =12kΩ and R3 =32kΩ, assuming the bridge is at balance.

Sol. Rx= R2*R3/R1=

Q2:If the resistors of the bridge shown in Fig.5-2 are as follows: R1 =1kΩ, R2=1.6kΩ, R3=3.5kΩ and R4=7.5kΩ. Is the bridge balance? If not then calculate the current through the galvanometer.(E=10v and Rm=200Ω)

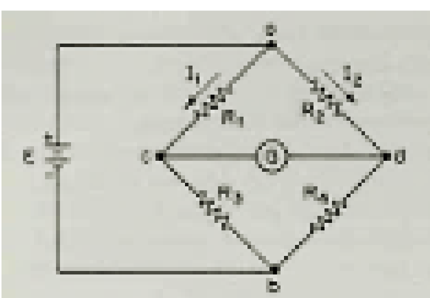
Sol. It is not balance since R1*R4 ≠ R2*R3

= + We must find first Rth =

And then find Eth= E(R1/(R1+R3) - R2/(R2+R4))

Im =

Q3: The bridge shown below has R1=100Ω, R2=100Ω and R4=50Ω. The G has a resistance of 200Ω . If the current in the G is equal to 0.5mA, calculate the value of the unknown resistance R3. Take E=60V.



Sol. 60= 100 I1 + (I1 - 0.0005) *R3 ------1

60= 100I2 + (I2 + 0.0005)*50 -------2

Vc = 60 – 100*I1 also Vd = 60 – 100*I2

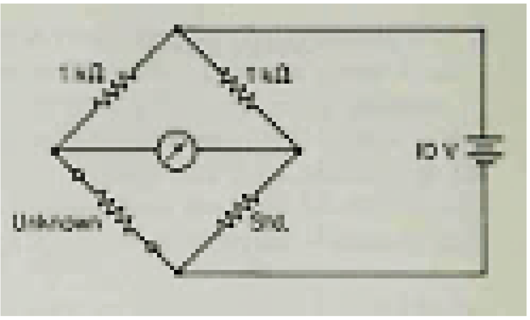
Vc – Vd =100*I2 - 100* I1 = 0.0005*Rm = 0.1V so I2 = 0.001 + I1 , substitute in eq.2 and solving for I1 yield

60 = 100 (0.001 + I1) + (0.001 + I1 + 0.0005) *50

I1 = 0.3988 Amp. Substitute in eq.1 yields

60 = 100*0.3988 + (0.3988 – 0.0005)*R3, from which R3= 50.5Ω

Q4: The standard resistance arm of the bridge shown below has a range from 0 to 100Ω with a resolution of 0.001Ω. The G has an internal resistance of 100Ω and can be read to 0.5µA. When the unknown resistance is 50Ω, what is the resolution of the bridge in both ohms and per cent of the unknown?



AC Bridges:

AC bridges are used to measure inductance and capacitance. The power source is an AC voltage at a desired frequency. The detector can be a headphone or oscilloscope. The bridge is shown in Fig.5-4. A complex notation for the impedances is used. The balance condition requires that the voltage difference from A to C be zero. This will be the case when the voltage drop from B to A equals the voltage drop from B to C, in both magnitude and phase.

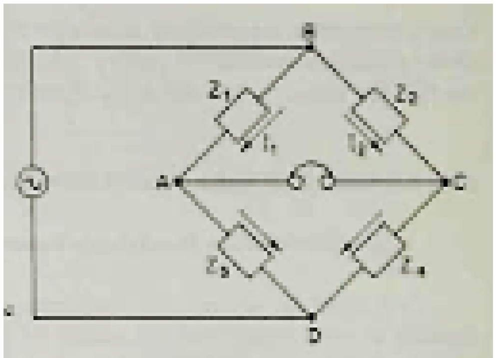


Fig.5-4 An AC Bridge.

EBA = EBC or I1 *Z1 = I2*Z2

For zero detector current (the balance condition), the current are

and substitute in the above eqs. Yield

Z1*Z4 = Z2*Z3 or using admittance instead of impedances, Y1*Y4 = Y2*Y3

The above impedance equation states that the product of the impedances of one pair of opposite arms equal the product of impedances of the other pair of opposite arms,

with the impedances expressed in complex notation. If the impedances is written in the form Z = magnitude and angle (Z = M<Ɵ), where M represents the magnitude and Ɵ the phase angle of the complex impedances, the impedance eq. Can be rewritten in the form

(M1<Ɵ1) *(M4<Ɵ4) = (M2<Ɵ2)* (M3<Ɵ3)

Since in multiplication of complex numbers the magnitudes are multiplied and the phase angles added, then

M1 M4 <(Ɵ1+Ɵ4) = M2 M3 <(Ɵ2+Ɵ3)

The above last eq. shows that the two conditions must be met, simultaneously when balancing an AC bridge. The first condition is that the magnitudes of the impedances satisfy the relationship

M1 M4 = M2 M3

That *is the product of the magnitudes of the opposite arms must be equal*

The second condition requires that the phase angles of the impedances satisfy the relationship

<Ɵ1 + <Ɵ4 = <Ɵ2 + <Ɵ3

And in words *the sum of the phase angles of the opposite arms must be equal.*

Example:

The impedances of the ac bridge of Fig.5-4 are given as follow:

Z1 = 100Ω < 80º (inductive impedance) ; Z2 = 250Ω (pure resistive)

Z3 = 400Ω <30º (inductive impedance) : Z4 = unknown = Zx

Determine at balance the constants of the unknown arm.

Sol. the first condition for bridge balance require that Z1 Z4 = Z2 Z3

= 1000Ω Z4 = Z2 Z3/ Z1 =

The second condition for bridge balance require that Ɵ1 + Ɵ4 = Ɵ2 + Ɵ3

Ɵ4 = Ɵ2 + Ɵ3 - Ɵ1 = 0 + 30 – 80 = -50º

Hence the unknown impedance Z4 = 1000<-50º (capacitive impedance).

Example: The bridge of fig. 5-4 is in balance with the following constants: arm BC, R=450Ω: arm AB, R=300Ω in series with C=0.265 µF; arm CD, unknown; arm DA, R=200Ω in series with L=15.9mH. The oscillator frequency is 1KHz. Find the constant of arm CD.

Sol. the general equation for bridge balance is Z1 Z4 = Z2 Z3

, Z1 = R – j/wc =(300 – j600) Ω , Z2 = R = 450Ω, Z3 = R + jw*L* = (200 + j100)Ω.

= +j150Ω Z4 = Z2Z3/Z1=

Which is a pure inductance. So XL = 150 = W*L* = 2лf*L* and solving for *L* we obtain *L*=23.9 mH.

-Notes: 1. If the impedance (Z) of any arm consists of only resistance (R) then Z=R.

2. If the impedance (Z) consists of only capacitance (C) then 3. If the impedance (Z) consists of R in series with C then

4. if the impedance (Z) consists of R in parallel with C then it is simpler to use the admittance (Y) instead of impedance and

5. If the impedance (Z) consists of R in series with the inductance (L), then

And note that *w* is the angular frequency of the source in rad./sec.= 2лf

- Comparison Bridges:

a. Capacitance Comparison Bridge:

A basic capacitance comparison bridge is shown in Fig.5-5. The ratio arms are both resistive and are represented by R1 and R2. The standard arm consists of capacitor Cs in series with resistor Rs. Cx represents the unknown capacitor and Rx is the leakage resistance of the capacitor.

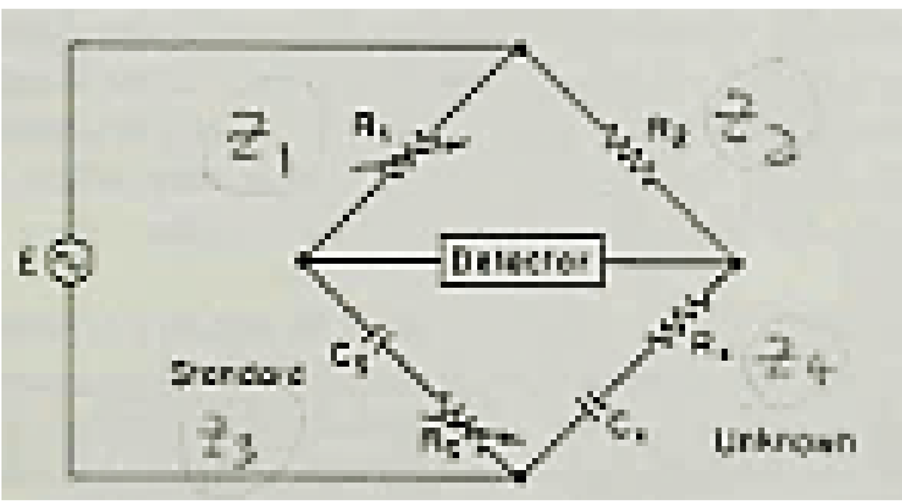


Fig.5-5 Capacitance Comparison bridge.

Z1 = R1 : Z2 = R2 : Zs = Rs – j/wCs **:** Z4 = Rx – j/wCx

Subs. In the general equation for bridge balance, we obtain

$$R1\left(Rx - \frac{j}{wcx}\right) = R2\left(Rs - \frac{j}{wcs}\right)$$

$$R1 * Rx - R1\frac{j}{wcx} = R2 * Rs - R2\frac{j}{wcs}$$

Two complex numbers are equal when both their real terms and their imaginary terms are equal. So equating the real term

R1* Rx = R2 *Rs or Rx = Rs * R2/R1 [same as Wheatstone bridge]

Equating the imaginary terms

jR1/wCx = jR2/wCs or Cx = Cs * R1/R2

b. Inductance Comparison Bridge:

The bridge is shown in Fig.5-6, the unknown inductance is determined by comparison with a known standard inductor. The derivation of the balance

equations is same as for the capacitance comparison bridge. It can be shown that the inductive balance equation yields

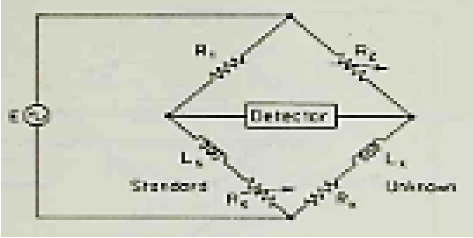
And the resistive balance equation gives

[same as in Wheatstone bridge]

Fig.5-6 Inductance Comparison bridge

- MAXWELL Bridge:

The Maxwell



- MAXWELL Bridge:

The Maxwell Bridge whose diagram is shown in Fig.5-7, measures an unknown inductance in terms of a known capacitance. One of the ratio arms has a resistor and a capacitor in parallel, so it is easier to write the balance equations using admittance of arm 1 instead of impedance.

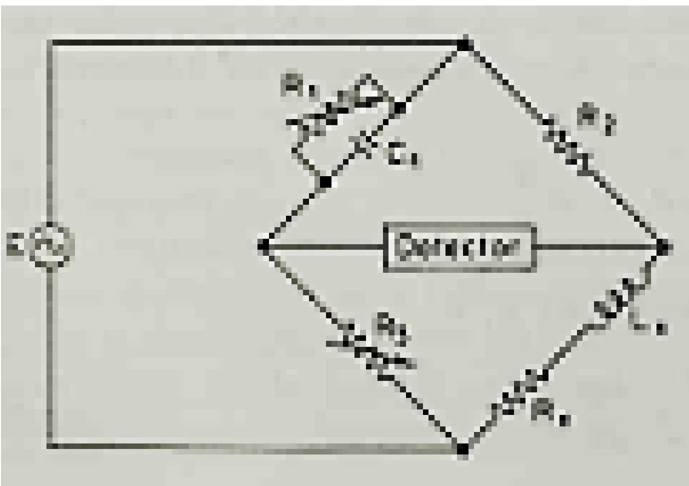


Fig.5-7 Maxwell Bridge for inductance measurements.

So Zx = Z2 *Z3 *Y1 ---1 , where Y1 is the admittance of arm 1. Reference to Fig5-7

Z2 = R2 Z3 = R3 and Y1 = 1/R1 + jwC1

Subs. In eq.1 yields

Separation of the real and imaginary terms yields

Rx = R2*R3 /R1 and Lx = R2*R3*C1

PROBLEMS:

Note: The bridge configuration shown below in Fig.5-8 is used in all questions.

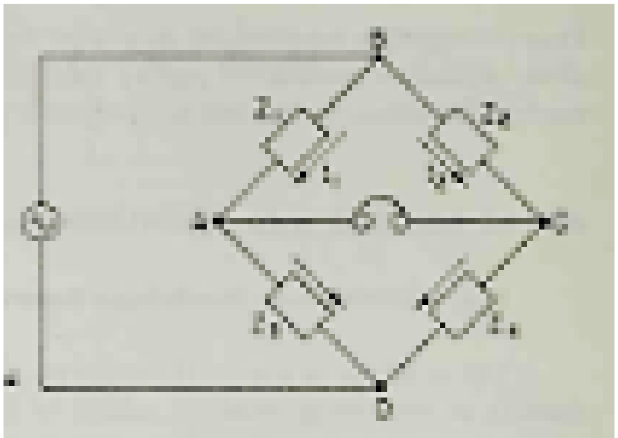


Fig.5-8 Bridge configuration

Q1: A balance AC bridge has the following constants: arm AB, R=2000Ω in parallel with C=0.047µF; arm BC, R=1000Ω in series with C=0.47µF; arm CD unknown; arm DA, C=0.5µF. The frequency of oscillation is 1000Hz. Find the constant of arm CD.

Q2: A bridge is balanced at 1000Hz and the following constants: AB, 0.2µF pure capacitance; BC, 500Ω pure resistance; CD, unknown; DA, R=100Ω in parallel with C=0.1µF. Find the R and C or L constants of arm CD, considered as a series circuit.

Q3: A 1000Hz bridge has the following constants: arm AB, R=1000Ω in parallel with C=0.5µF; BC, R=1000Ω in series with C=0.5µF; CD, L=30mH in series with R=200Ω. Find the constant of arm DA to balance the bridge.

Q4: An AC bridge has the following constants: arm AB, R=1000Ω in parallel with C=0.159µF; BC, R=1000Ω; CD, R=500Ω; DA, C=0.636µF in series with an unknown resistance. Find the value of the resistance in arm DA to produce balance. Then find the frequency for which the bridge is in balance.

Q5: An AC bridge has in arm AB a pure capacitance of 0.2µF, in arm BC, a pure resistance of 500Ω, a series combination of R=50Ω and L=0.1H. Arm DA consist of a capacitor C=2µF in series with a variable resistance Rs , w=5000 rad./sec. Find the value of Rs to obtain bridge balance.

Q6: The arms of the bridge are arranged, for balance, as follows: arm AB consists of an imperfect C1 with shunt resistance R1 representing the loss; BC and CD of resistances of 1000Ω each; DA of standard capacitance of 0.0115µF in series with a 140Ω resistance. Find the value of C1 and R1. The supply frequency is 7500 rad./sec.

Solutions:

Q1: Y1= 1/R1 + jwC1 = 1/2000 + j 0.0003 ; Z2= 1000 – j338.6 ; Z4=-j318

$$Zx = Z4 = Z2 * Z3 * Y1$$

$$Zx = (1000 - j338.6)(-j318)\left(\frac{1}{2000} + j0.0003\right)$$

$$Zx = 41.6 - j191.3$$

So 1/wC4=191.3 and C4=0.83µF ; R4=41.6Ω

Q2: Z1= 1/jwC1 = -j795.7Ω ; Z2=500Ω ; Y3= 1/300 +jwC3=1/300+j0.000628

$$Zx\ Z1 = Z2*1/Y3 = \frac{500}{-j795.7(\frac{1}{300}+j0.000628)} = \frac{500}{0.5-j2.65}$$

$$= \frac{500}{2.7\angle -79.2} = 185\angle 79.3 = 34.3 + j181$$

So $2\pi fL = 181$; L=29mH

Q3: Y1=1/R1 + jwC1 =0.001+ j0.00314

Z2 = 1000 – j/wC2 = 1000 – j318.5 ; Z3=200 +jwL=200 + j188.5

$$Zx = \frac{200+j188.5}{(1000-j318.5)(0.001+j0.00314)} = \frac{274.8<43.3}{2+j2.8} = \frac{274.8<43.3}{3.458<54.4}$$

$$= 79.2 < -11.16 = 77.1 - j15.33$$

1/wC3=15.33 ; C3= 10.38µF

Q4: Z2*Z3*Y1=Z4

$$1000\left(Rx + \frac{1}{jw(0.636*10^{-6})}\right)\left(\frac{1}{1000} + jw(0.159)*10^{-6}\right) = 500$$

$$0.002Rx + 2jwRx(0.159)*10^{-6} + \frac{0.002}{jw(0.636*10^{\wedge}-6)} + 0.5=1$$

Equate real parts yield 0.002Rx + 0.5 = 1 ; Rx = 250Ω

Equate imaginary parts and solving for w yields

$$jwRx(0.318)*10^{-6} = -\frac{0.002}{jw(0.636)*10-6} = j\frac{0.002}{w(0.636*10^{\wedge}-6)}$$

$$w^2 = 0.002/(250(0.318*10^{-6})(0.636*10^{-6}))$$

$w = 2\pi f$ From which f=1000 Hz

Q5: $\frac{1}{wC1} = \frac{1}{5000*0.2*10^{-6}} = 1000\Omega$

$\frac{1}{wC3} = \frac{1}{5000*2*10^{\wedge}-6}$=100Ω

Z1 Z4 = Z2 Z3 ; Z1= 1/jwC1 = -j1000 ; Z4 =50+jwL=50+j500

$$-j1000(50 + j500) = 500(Rs - j100)$$

$$-j50000 + 500000 = 500Rs - j50000$$

Equating the real parts and solving for Rs yields Rs=1000Ω

Q6: Y1=1/R1 +jwC1 ; Z2=1000Ω ; Z4=1000Ω ; Z3=140-j/wC3=140-j11594

Z4 = Z2 Z3 Y1