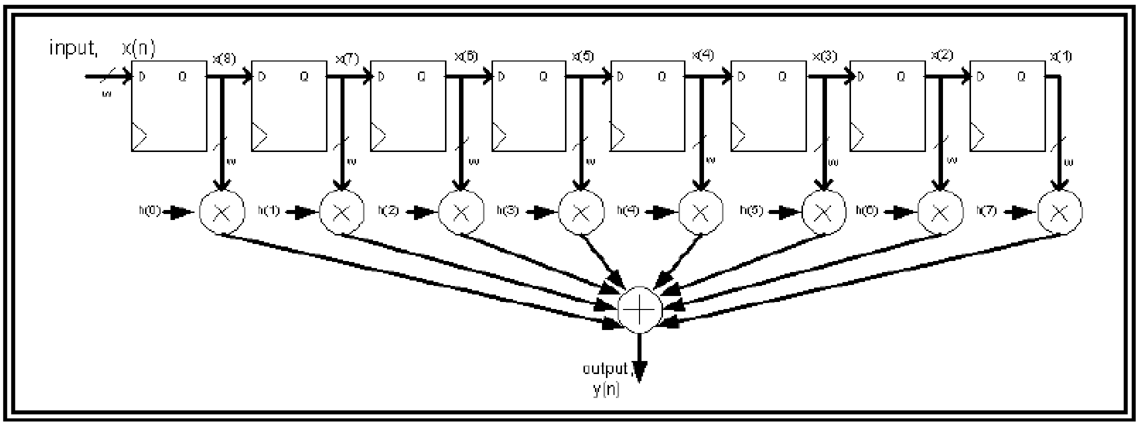
Ministry of Higher Education and Scientific Research
Middle Technical University
Electrical Engineering Technical College

Training package in

Digital Signal Processing
(Discrete Time Systems)

For

Students of third class
Control and Automation Engineering Techniques



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Discrete Time Systems

1/ Overview

1 / A –Target population :-

For students of third class

Department of Medical Instrumentation Eng. Techniques

1 / B –Rationale :-

This unit introduces discrete time systems and focus on representation of these systems and properties such as linearity, time invariance, and causality.

1 / C –Central Idea :-

The major topics discussed in this unit are included in the following outline.

- Discrete Time systems
- Input – output Description of Systems
- Block Diagram representation of Discrete – Time Systems
- Classification of Discrete – Time Systems

2/ Performance Objectives :-

After studying the 3rd modular unit, the student will be able to:-

- Identify discrete time systems.
- Describe discrete time systems.
- Represent discrete time systems by block diagram.
- Classify discrete time systems.

3/ Pre test :-

Circle the correct answer :-

1. **Discrete time system in general is a system dedicated for processing:-**
   a- Analog signals before converting to discrete time signals.
   b- Analog signals after converting to discrete time signals.
   c- Any discrete time signal.
   d- Digital signals.

2. **Digital processor is:**
   a- Part of discrete time systems
   b- Another name of discrete time systems.

c- Any of above

d- Non of above

3. Discrete-time systems can be realized by:-

a- Hardware components.

b- Software programs.

c- Combination of above.

d- Any of above.

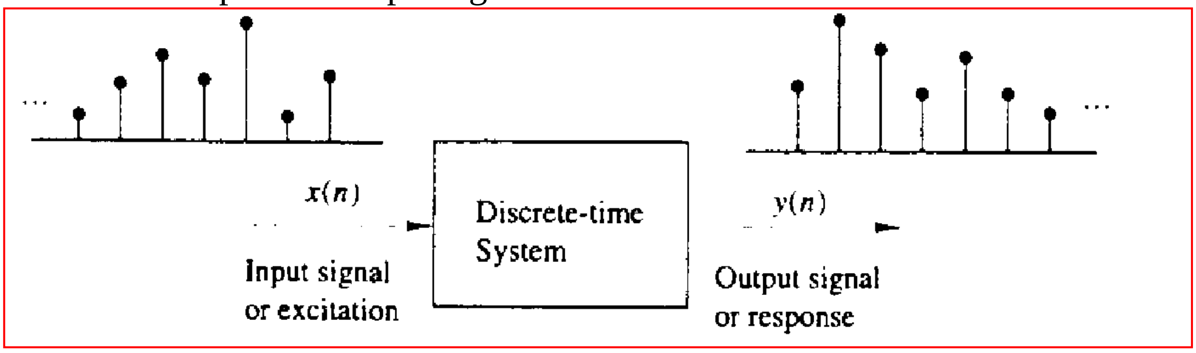
4/ the text :-

Discrete – Time Systems

In many applications of digital signal processing we wish to design a device or an algorithm that performs some prescribed operation on discrete – time signal. Such a device or algorithm is called a discrete – time system. More specifically, a discrete – time system is a device or algorithm that operates on a discrete – time signal called the input or excitation, according to some well – defined rule, to produce another discrete – time signal called the output or response of the system.

Input – output Description of Systems

The input – output description of a discrete – time system consists of a mathematical expression or a rule, which explicitly defines the relation between the input and output signals.



Block diagram representation of a discrete – time system

Example

Determine the response of the following systems to the input signal

$$x(n) = \begin{cases} |n|, & -3 \le n \le 3 \\ 0, & otherwise \end{cases}$$

a) $y(n) = x(n)$

b) $y(n) = x(n-1)$

c) $y(n) = x(n+1)$

d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

Solution First, we determine explicitly the sample values of the input signal

$$x(n) = \{\ldots\ldots 0,3,2,1,0,1,2,3,0 \ldots\ldots\}$$

Next, we determine the output of each system using its input – output relationship.

a) In this case the output is exactly the same as the input signal. Such a system is known as the identity system.

b) This system simply delays the input by one sample

$x(n)$	3	2	1	0	1	2	3
$x(n-1)$	0	3	2	1	0	1	2

c) In this case the system "advances" the input one sample into the future.

$x(n)$	3	2	1	0	1	2	3
$x(n+1)$	2	1	0	1	2	3	0

d) The output of this system at any time is the mean value of the present, the immediate past, and the immediate future samples

Example

The accumulator described by

$$y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \cdots$$

Is excited by the sequence $x(n) = nu(n)$. Determine its output under the condition that:

a) It is initially relaxed [i.e.,$y(-1) = 0$].

b) Initially, $y(-1) = 1$.

Solution The output of the system is defined as

$$y(n) = \sum_{k=-\infty}^{n} x(k) = \sum_{k=-\infty}^{-1} x(k) + \sum_{k=0}^{n} x(k)$$

$$= y(-1) + \sum_{k=0}^{n} x(k)$$

But

$$\sum_{k=0}^{n} x(k) = \frac{n(n+1)}{2}$$

For a

$$y(n) = \frac{n(n+1)}{2}$$

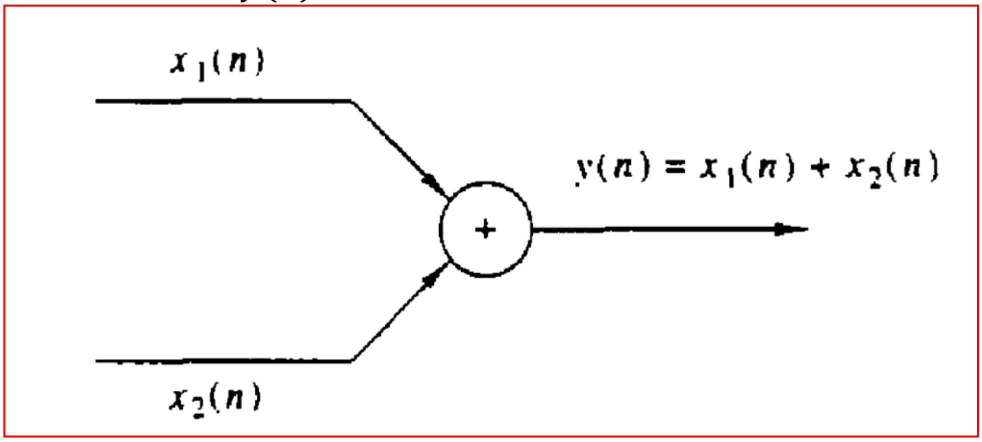
For b

$$y(n) = 1 + \frac{n(n+1)}{2}$$

Block Diagram representation of Discrete – Time Systems

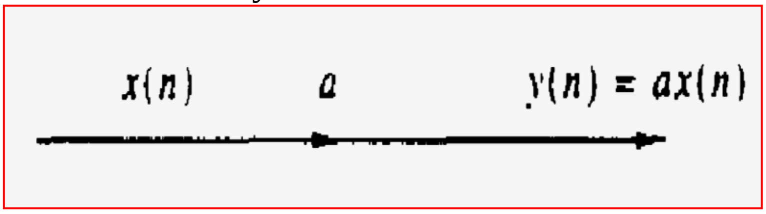
It is useful at this point to introduce a block diagram representation of discrete – time systems. For this purpose we need to define some basic building blocks that can be interconnected to form complex systems.

An adder. The figure below illustrate a system (adder) that performs the addition of two signal sequences to form another (the sum) sequence, which we denote as $y(n)$.

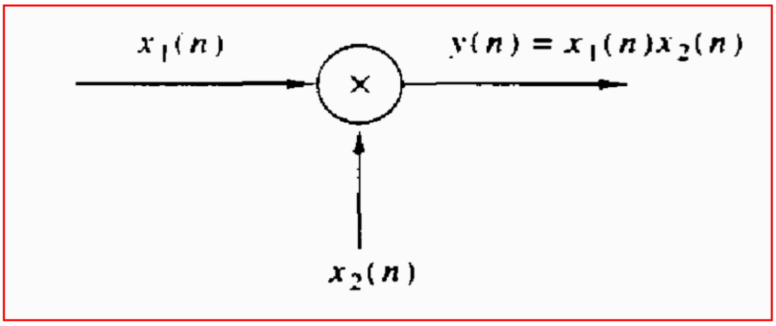


Note that it is not necessary to store either one of the sequences in order to perform the addition. In other words, the addition operation is memoryless.

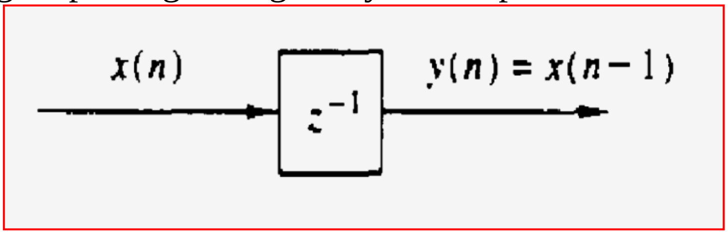
A constant multiplier. This operation is depicted by figure below, and simply represent applying a scale factor on the input $x(n)$. Note that this operation is also memoryless.



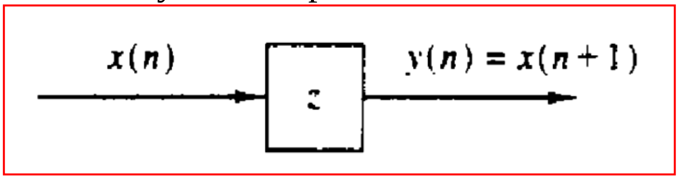
A signal multiplier. Figure below illustrates the multiplication of two signal sequences, the multiplication operation is memoryless.



A unit delay element. The unit delay is a special system that simply delays the signal passing through it by on sample.



A unit advance element. In contrast to the unit delay, a unit advance moves the input a head by one sample.

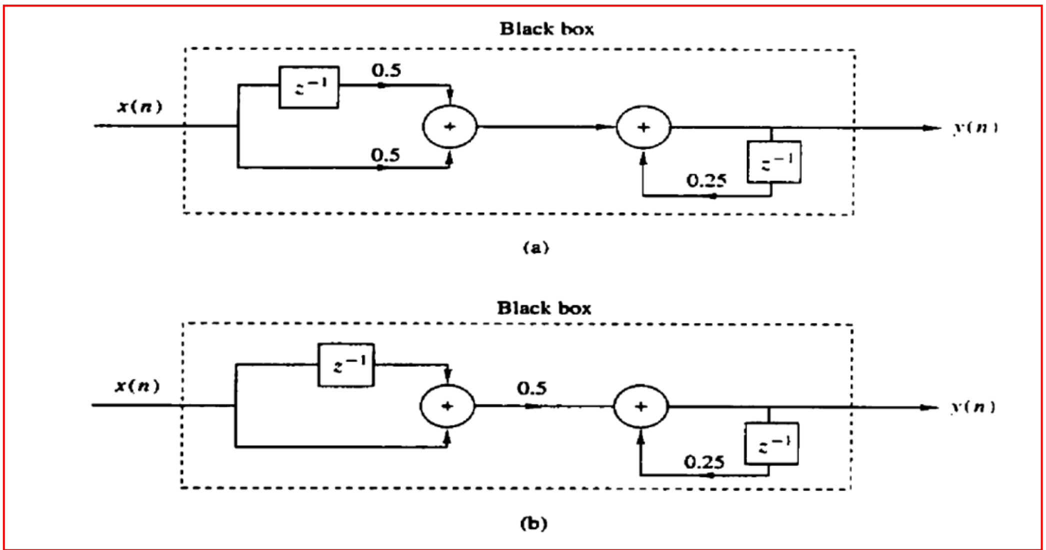


Example:-

Using basic building blocks introduced above, sketch the block diagram representation of the discrete – time system described by the input – output relation.

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

Solution:-



Classification of Discrete – Time Systems:-

a) **Static versus dynamic systems.**

Static systems	Dynamic systems
$\boldsymbol{y(n) = ax(n)}$ $\boldsymbol{y(n) = nx(n) + bx^3(n)}$	$\boldsymbol{y(n) = x(n) + 3x(n-1)}$ $\boldsymbol{y(n) = \sum_{k=0}^{n} x(n-k)}$

b) **Time – Invariant versus Time – Variant systems**

A relaxed system T is time invariant or shift invariant if and only if

$$x(n) \xrightarrow{T} y(n)$$

Implies that

$$x(n-k) \xrightarrow{T} y(n-k)$$

For every input signal $x(n)$ and every time shift k.

In general, we can write the output as

$$y(n,k) = T[x(n-k)]$$

Now if this output $y(n,k) = y(n-k)$, for all possible values of k, the system is time invariant. On the other hand, if the output $y(n,k) \neq y(n-k)$, even for one value of k, the system is time variant.

Example:-

The system described by the input – output equation

$$y(n) = T[x(n)] = x(n) - x(n-1)$$

Now if the input is delayed by k units in time and applied to the system

$$y(n,k) = x(n-k) - x(n-k-1)$$

On the other hand, if we delay $y(n)$ by k units in time, we obtain

$$y(n-k) = x(n-k) - x(n-k-1)$$

Therefore, $y(n,k) = y(n-k)$ and the system is time invariant.

Example:-

The system described by the input – output equation

$$y(n) = T[x(n)] = x(-n)$$

The response of this system to $x(n-k)$ is

$$y(n,k) = T[x(n-k)] = x(-n-k)$$

Now, if we delay the output $y(n)$ by k units in time, the result will be

$$y(n-k) = x(-n+k)$$

Since $y(n,k) \neq y(n-k)$, the system is time variant.

c) Linear versus nonlinear systems.

A relaxed T system is linear if and only if

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

For any arbitrary input sequences $x_1(n)$ and $x_2(n)$, and any arbitrary constants a_1 and a_2.

Example:-

Determine if the systems described by the following input – output equations are linear or nonlinear.

a) $y(n) = nx(n)$

b) $y(n) = x^2(n)$

Solution:-

a) For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$y_1(n) = nx_1(n)$$

$$y_2(n) = nx_2(n)$$

A linear combination of the two input sequences results in the output

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)]$$

A linear combination of the $y_1(n)$ and $y_2(n)$ results in

$$a_1y_1(n) + a_2y_2(n) = n[a_1x_1(n) + a_2x_2(n)]$$

Since $y_3(n) \equiv a_1y_1(n) + a_2y_2(n)$ the system is linear.

b) The responses of the system to two separate input signals are

$$y_1(n) = {x_1}^2(n)$$

$$y_2(n) = {x_2}^2(n)$$

The response of the system to a linear combination of these two input signals is

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$$

$$= [a_1x_1(n) + a_2x_2(n)]^2$$

$$= {a_1}^2{x_1}^2(n) + 2a_1a_2x_1(n)x_2(n) + {a_2}^2{x_2}^2(n)$$

On the other hand, if the system is linear, it would produce a linear combination of the two outputs

$$a_1y_1(n) + a_2y_2(n) = a_1{x_1}^2(n) + a_2{x_2}^2(n)$$

Since the actual output of the system, is not equal to the above equation, the system is nonlinear.

c) Causal versus noncausal systems

In mathematical terms, the output of a causal system satisfies an equation of the form

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots]$$

Where F[.] is some arbitrary function.

If a system does not satisfy this definition, it is called noncausal. Such a system has an output that depends not only on present and past inputs but also on future inputs.

Example:-

Determine if the systems described by the following input – output equations are causal or noncausal.

a) $y(n) = x(n) - x(n-1)$ b) $y(n) = \sum_{k=-\infty}^{n} x(k)$

c) $y(n) = ax(n)$ d) $y(n) = x(n) + 3x(n+4)$

d) $y(n) = x(n^2)$ f) $y(n) = x(2n)$

g) $y(n) = x(-n)$

Solution:-

The systems described by parts (a), (b), and (c) are causal.

The systems described by rest parts are noncausal.

d) Stable versus unstable systems

An arbitrary relaxed system is said to be bounded input – bounded output (BIBO) stable if and only if every bounded input produces a bounded output.

The conditions that the input sequence $x(n)$ and the output sequence $y(n)$ are bounded is translated mathematically to mean that there exist some finite numbers,

Say M_x and M_y such that

$$|x(n)| \le M_x < \infty \qquad |y(n)| \le M_y < \infty$$

For all n. If, for some bounded input sequence $x(n)$, the output is unbounded (infinite), the system is classified as unstable.

Example:-

Consider the nonlinear system described by the input – output equation

$$y(n) = y^2(n-1) + x(n)$$

As an input sequence we select the bounded signal

$$x(n) = C\delta(n)$$

Where C is a constant. We also assume that $y(-1) = 0$. Then the output sequence is

$$y(0) = C, \qquad y(1) = C^2, \qquad \ldots\ldots y(n) = C^{2n}$$

Clearly, the output is unbounded when $1 < |C| < \infty$. Therefore, the system is unstable.

5/ Post test :-

1. The input – output description of a discrete – time system is
   a) A block diagram representation of the system.
   b) A mathematical expression or a rule representation of the system.
   c) Analog to digital conversion.
   d) Sampling of analog signal.
2. In static systems the output signal is function of:
   a) Current input signals.
   b) Previous input signals.
   c) Current and previous input signals.
   d) Disturbance.
3. Stable systems is:
   a) Bounded input bounded output.
   b) Unbounded input bounded output.
   c) Bounded input unbounded output.
   d) Unbounded input unbounded output.

Key Answers
Pre test:
1.b 2.a 3.d
Post test
1.b 2.a 3.a

7/References :-

1. Schaum's Outline of Theory and Problems of Digital Signal processing.
2. Digital signal processing, principles, algorithms, and applications by John G. Proakis and Dimitris G. Manolakis.
3. Signal and systems, Alan Oppenheim.