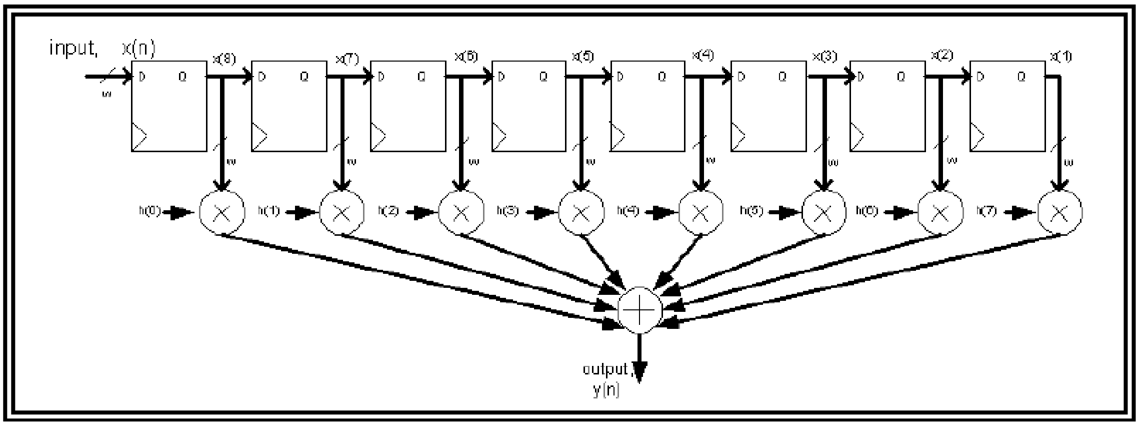
Ministry of Higher Education and Scientific Research
Middle Technical University
Electrical Engineering Technical College

Training package
in

Digital Signal Processing
(Image Processing III)

For

Students of third class
Control and Automation Engineering Techniques



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September/2021

Image Processing III

1/ Overview

1 / A –Target population :-

For students of third class

Department of Medical Instrumentation Eng. Techniques

1 / B –Rationale :-

This unit introduces Image processing techniques

1 / C –Central Idea :-

The major topics discussed in this unit are included in the following outline.

- **Image Spectra**
- **Image Compression by Discrete Cosine Transform**
- **Two-Dimensional Discrete Cosine Transform**

2/ Performance Objectives :-

After studying the 15th modular unit, the student will be able to know:-

1. Principles of image compression.

4/ the text :-

Image Spectra

In one-dimensional digital signal processing such as for speech and other audio, we need to examine the frequency contents, check filtering effects, and perform feature extraction. Image processing is similar. However, we need to apply a two-dimensional discrete Fourier transform (2D-DFT) instead of a one-dimensional (1D) DFT. The spectrum including the magnitude and phase is also in two dimensions. The equations of the 2D-DFT are given by:

$$X(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} p(m,n) W_M^{um} W_N^{vn},$$

where $W_M = e^{-j\frac{2\pi}{M}}$ and $W_N = e^{-j\frac{2\pi}{N}}$,
m and n = pixel locations
u and v = frequency indices.

Taking the absolute value of the 2D-DFT coefficients X(u,v) and dividing the absolute value by (M x N), we get the magnitude spectrum as

$$A(u,v) = \frac{1}{(N \times M)} |X(u,v)|.$$

Example

Determine the 2D-DFT coefficients and magnitude spectrum for the following 2x2 image:

$$\begin{bmatrix} 100 & 50 \\ 100 & -10 \end{bmatrix}.$$

Solution: Since M = N = 2,

$$X(u, v) = p(0, 0)e^{-j\frac{2\pi u\times 0}{2}} \times e^{-j\frac{2\pi v\times 0}{2}} + p(0, 1)e^{-j\frac{2\pi u\times 0}{2}} \times e^{-j\frac{2\pi v\times 1}{2}} + p(1, 0)e^{-j\frac{2\pi u\times 1}{2}} \times e^{-j\frac{2\pi v\times 0}{2}} + p(1, 1)e^{-j\frac{2\pi u\times 1}{2}} \times e^{-j\frac{2\pi v\times 1}{2}}.$$

For $u = 0$ and $v = 0$, we have

$$X(0, 0) = 100e^{-j0} \times e^{-j0} + 50e^{-j0} \times e^{-j0} + 100e^{-j0} \times e^{-j0} - 10e^{-j0} \times e^{-j0} = 100 + 50 + 100 - 10 = 240.$$

For $u = 0$ and $v = 1$, we have

$$X(0, 1) = 100e^{-j0} \times e^{-j0} + 50e^{-j0} \times e^{-j\pi} + 100e^{-j0} \times e^{-j0} - 10e^{-j0} \times e^{-j\pi} = 100 + 50 \times (-1) + 100 - 10 \times (-1) = 160.$$

Following similar operations,

$$X(1, 0) = 60 \quad \text{and} \quad X(1, 1) = -60.$$

Thus, we have DFT coefficients as

$$X(u, v) = \begin{bmatrix} 240 & 160 \\ 60 & -60 \end{bmatrix}.$$

We can calculate the magnitude spectrum as

$$A(u,v) = \begin{bmatrix} 60 & 40 \\ 15 & 15 \end{bmatrix}.$$

Image Compression by Discrete Cosine Transform

Image compression is a must in our modern media systems, such as digital still and video cameras and computer systems. The purpose of compression is to reduce information storage or transmission bandwidth without losing image quality or at least without losing it significantly. Image compression can be classified as lossless compression or lossy compression. Here we focus on lossy compression using discrete cosine transform (DCT).

The DCT is a core compression technology used in the industry standards JPEG (Joint Photographic Experts Group) for still-image compression and MPEG (Motion Picture Experts Group) for video compression, achieving compression ratios of 20:1 without noticeable quality degradation. JPEG standard image compression is used every day in real life.

Two-Dimensional Discrete Cosine Transform

Image compression uses 2D-DCT, whose transform pairs are defined as:

Forward DCT:

$$F(u, v) = \frac{2C(u)C(v)}{\sqrt{MN}} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} p(i, j) \cos\left(\frac{(2i+1)u\pi}{2M}\right) \cos\left(\frac{(2j+1)v\pi}{2N}\right)$$

Inverse DCT:

$$p(i, j) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \frac{2C(u)C(v)}{\sqrt{MN}} F(u, v) \cos\left(\frac{(2i+1)u\pi}{2M}\right) \cos\left(\frac{(2j+1)v\pi}{2N}\right)$$

where

$$C(m) = \begin{cases} \frac{\sqrt{2}}{2} & \text{if } m = 0 \\ 1 & \text{otherwise} \end{cases}$$

$p(i, j)$ = pixel level at the location (i, j)

$F(u, v)$ = DCT coefficient at the frequency indices (u,v).

JPEG divides an image into 8 x 8 image subblocks and applies DCT for each subblock individually. Hence, we simplify the general 2D-DCT in terms of 8 x 8 size. The equation for 2D 8 x 8 DCT is modified as:

$$F(u, v) = \frac{C(u)C(v)}{4} \sum_{i=0}^{7} \sum_{j=0}^{7} p(i, j) \cos\left(\frac{(2i+1)u\pi}{16}\right) \cos\left(\frac{(2j+1)v\pi}{16}\right).$$

The inverse of 2D 8 x 8 DCT is expressed as:

$$p(i, j) = \sum_{u=0}^{7} \sum_{v=0}^{7} \frac{C(u)C(v)}{4} F(u, v) \cos\left(\frac{(2i+1)u\pi}{16}\right) \cos\left(\frac{(2j+1)v\pi}{16}\right).$$

Example

Determine the 2D-DCT coefficients for the following image:

$$\begin{bmatrix} 100 & 50 \\ 100 & -10 \end{bmatrix}.$$

Solution: Applying N = 2 and M = 2

$$F(u, v) = \frac{2C(u)C(v)}{\sqrt{2 \times 2}} \sum_{i=0}^{1} \sum_{j=0}^{1} p(i, j) \cos\left(\frac{(2i+1)u\pi}{4}\right) \cos\left(\frac{(2j+1)v\pi}{4}\right).$$

For $u = 0$ and $v = 0$, we achieve:

$$F(0, 0) = c(0)c(0) \sum_{i=0}^{1} \sum_{j=0}^{1} p(i, j) \cos(0) \cos(0)$$

$$= \left(\frac{\sqrt{2}}{2}\right)^2 [p(0, 0) + p(0, 1) + p(1, 0) + p(1, 1)]$$

$$= \frac{1}{2}(100 + 50 + 100 - 10) = 120$$

For $u = 0$ and $v = 1$, we achieve:

$$F(0,1) = c(0)c(1) \sum_{i=0}^{1} \sum_{j=0}^{1} p(i,j) \cos(0) \cos\left(\frac{(2j+1)\pi}{4}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right) \times 1 \times \left(p(0,0)\cos\frac{\pi}{4} + p(0,1)\cos\frac{3\pi}{4} + p(1,0)\cos\frac{\pi}{4} + p(1,1)\cos\frac{3\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2}\left(100 \times \frac{\sqrt{2}}{2} + 50\left(-\frac{\sqrt{2}}{2}\right) + 100 \times \frac{\sqrt{2}}{2} - 10\left(-\frac{\sqrt{2}}{2}\right)\right) = 80$$

Similarly,

$$F(1,0) = 30 \quad \text{and} \quad F(1,1) = -30.$$

Finally, we get

$$F(u, v) = \begin{bmatrix} 120 & 80 \\ 30 & -30 \end{bmatrix}.$$

Example

Given the following DCT coefficients from a 2 x 2 image:

$$F(u, v) = \begin{bmatrix} 120 & 80 \\ 30 & -30 \end{bmatrix},$$

Determine the pixel p(0, 0).

Solution:

The inverse 2D-DCT with N = M = 2, i = 0, and j = 0, it follows that

$$p(0,0) = \sum_{u=0}^{1} \sum_{v=0}^{1} c(u)c(v)F(u,v)\cos\left(\frac{u\pi}{4}\right)\cos\left(\frac{v\pi}{4}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right) \times \left(\frac{\sqrt{2}}{2}\right) \times F(0,0) + \left(\frac{\sqrt{2}}{2}\right) \times F(0,1) \times \left(\frac{\sqrt{2}}{2}\right)$$

$$+ \left(\frac{\sqrt{2}}{2}\right) \times F(1,0) \times \left(\frac{\sqrt{2}}{2}\right) + F(0,1)\left(\frac{\sqrt{2}}{2}\right) \times \left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{1}{2} \times 120 + \frac{1}{2} \times 80 + \frac{1}{2} \times 30 + \frac{1}{2}(-30) = 100$$

7/References :-

1. Schaum's Outline of Theory and Problems ofDigital Signal processing.
2. Digital signal processing, principles, algorithms, and applications by John G. Proakis and Dimitris G. Manolakis.
3. Signal and systems, Alan Oppenheim.