Classical methods

Introduction

Frequency response, step response, impulse response and correlation method are called classical methods, since they are well proved methods. They are classified as off line methods because special inputs rather than normal operation inputs are used.

The measurement procedure consists of two steps:

1- Letting the process to settle into an equilibrium state of operating conditions of interest, and

2- Applying the test signal and recording the process response.

Usually the measurable o/p signal will contain additional noise, then the experiment must be planned in such away that the signal to noise ratio (S/N) is not too small. This can be done by enlarging the amplitude of the I/P signal, however nonlinearities in the process can not be avoided.

Taking several step responses and calculating the average from them may help to diminish the effect of random noise.

A- Step response identification

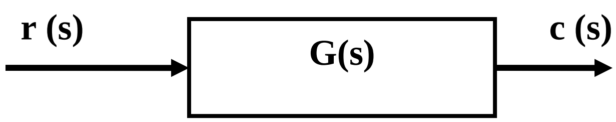
The step response of a system is most frequently used to analyze systems, and there is a large amount of terminology involved with step response. When exposed to a sudden change in the input, the system will initially have undesirable output period known as transient response. The steady state response of the system is the response after the transient response has ended.

Transient response plots show the value of the step response on the vertical axis. The horizontal axis is in units of time you specified for the data used to estimate the model.

Analysis of step response

Consider a system with T.F, G(s)

$$G(s) = \frac{c(s)}{r(s)}$$



The step response of any linear system becomes

$$c(s) = \frac{G(s)}{s}$$

A.1- First order system

To each unit of a system, which has a transfer function of the form

$$G(s) = \frac{k}{1 + T s}$$

A step input is applied and the output response is recorded. The point at which the output has reached 63.2% of its final value yields the value of T.F. Another method is to extend the initial slope of the output response until it intersects the line representing the steady state value. The point of intersection yields the value of ***T***.

- ***For a unit step input***

$$G(s) = \frac{k}{s\,(1+Ts)}$$

By taking the laplace inverse, the output in time domain

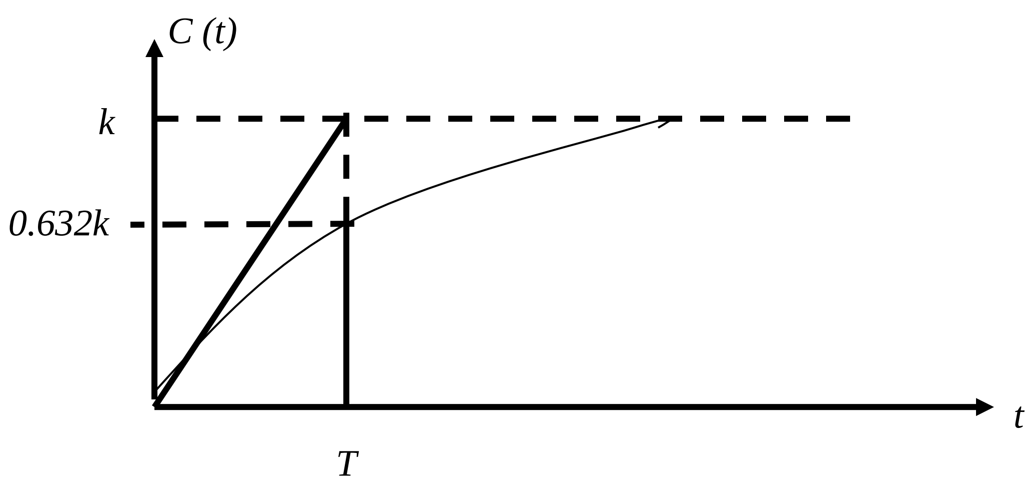
$$C(t) = k\,(1 - e^{-(t/T)})$$

- ***At t = T***

$$C(t) = k\,(1-e^{-(t/T)}) = k\,(1-e^{-1}) = k\,(1-.367) = 0.632*k$$

- ***At t = ∞***

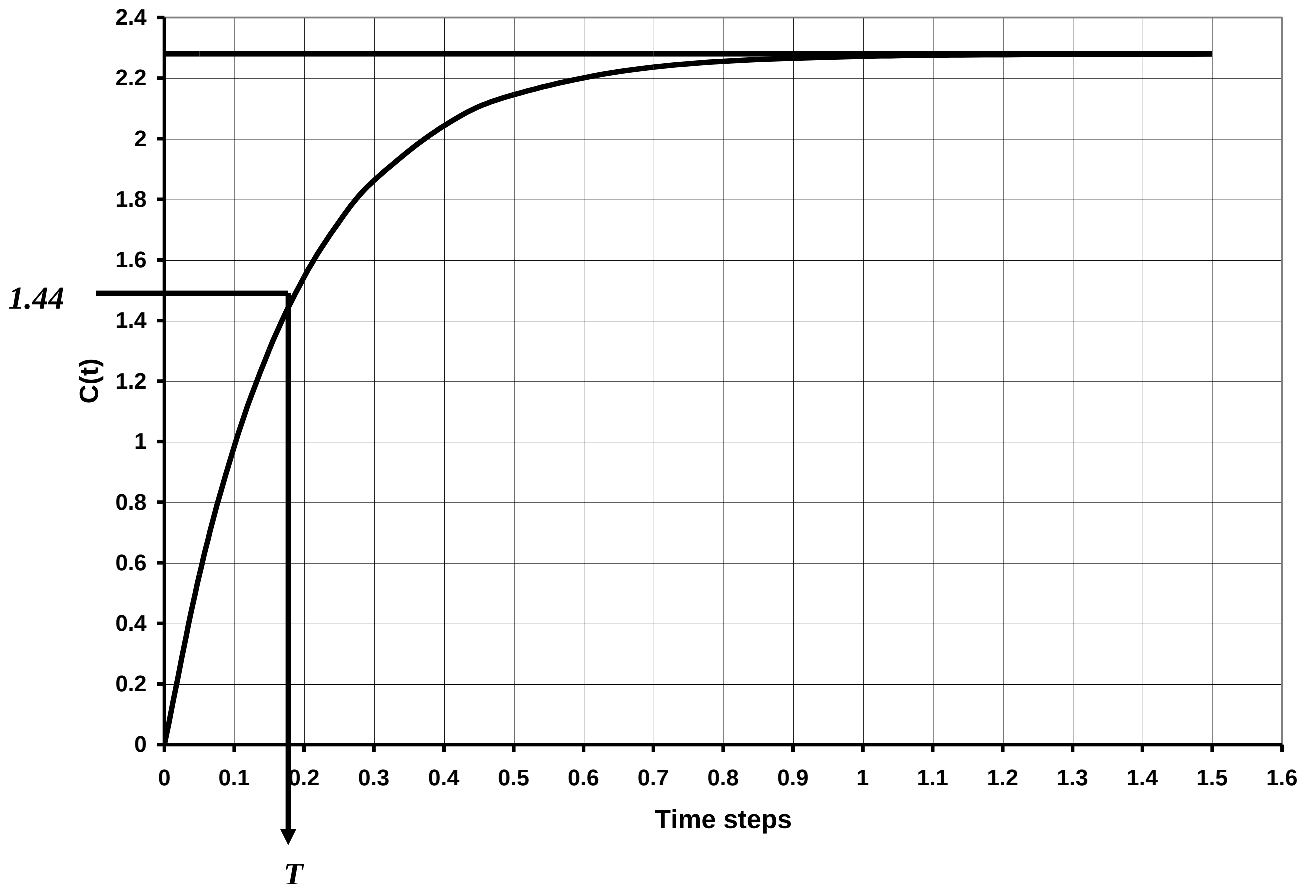
$$C(t) = k\,(1- e^{-\infty}) = k$$



But generally, gain is calculated by dividing the steady state output by the constant value of input step. **(Prove)**

Example: The samples of step response of first order system are given below

t	0	.05	.1	.15	.2	.25	.3	.4	.5	.7	1	1.5	2
C(t)	0	.561	.984	1.303	1.543	1.725	1.861	2.042	2.145	2.236	2.272	2.28	2.28



<u>Solution:</u>

- Gain= O/P s.s ÷ I/P = 2.28 ÷ 1 =2.28
- *To find T*

0.63* k = 1.44 , T = 0.18

- ***G (S) = 2.28 / (1+0.18 S)***

<u>Homework</u>:

<u>Q</u> For the following first order transfer function, illustrate the manner in which the time constant can be determined.

$$G(s) = \frac{k\,T\,s}{(T\,s+1)}$$

$$G(s) = \frac{k\,(1+T\,s)}{(\alpha\,T\,s+1)} \quad \text{and } \alpha < 1$$

Pure time delay

If any response is delayed by delay time, **τ** after the application of the step, it is assumed the system has a pure time delay term given in laplace form, exp **(-τs)**. The response is given by:

$$c(s) = \frac{k * e^{-\tau s}}{s(Ts+1)}$$

Or in time domain

$$C(t) = \begin{cases} 0 & t < \tau \\ k\left(1 - e^{-(t-\tau)/T}\right) & t \geq \tau \end{cases}$$

A-2 Periodic second order system

A periodic (over damped and critically damped only i.e. non oscillatory).

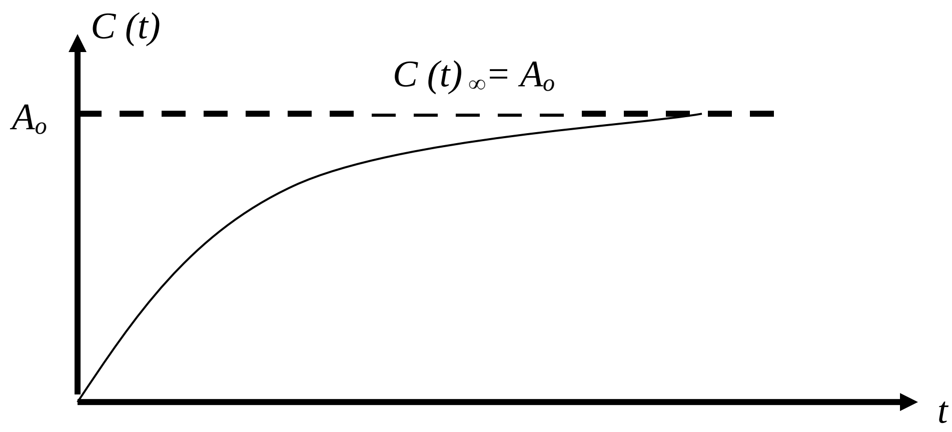
Slope-intercept method (Percent incomplete)

The time constants of a unit that has a transfer function

$$G(s) = \frac{k}{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}$$

where $T_1 > T_2$ can be experimentally determined by utilizing step input function. The output is given by:

$$C(t) = A_o - A_1 * e^{-t/T_1} + A_2 * e^{-t/T_2} \qquad \text{with } r(s) = \frac{1}{s}$$



)** The final value when ***t = ∞

$$C(\infty) = A_o - A_1 * e^{-\infty} + A_2 * e^{-\infty} = A_o$$

The output at steady state ***C(t)S.S*** = ***A_o***

***)** From the above figure, data can be obtained to plot the transient component versus time on semi log paper.

$$A_o - C(t) = A_1 * e^{-t/T_1} - A_2 * e^{-t/T_2}$$

***)** The graphical determination of T_1 and T_2 requires that the two time constants be appreciably different.

***)** If this condition is satisfied, then for $t \geq T_2$, the component $A_2 * e^{-t/T_2}$ becomes insignificant

$$A_o - C(t) \cong A_1 * e^{-t/T_1}$$

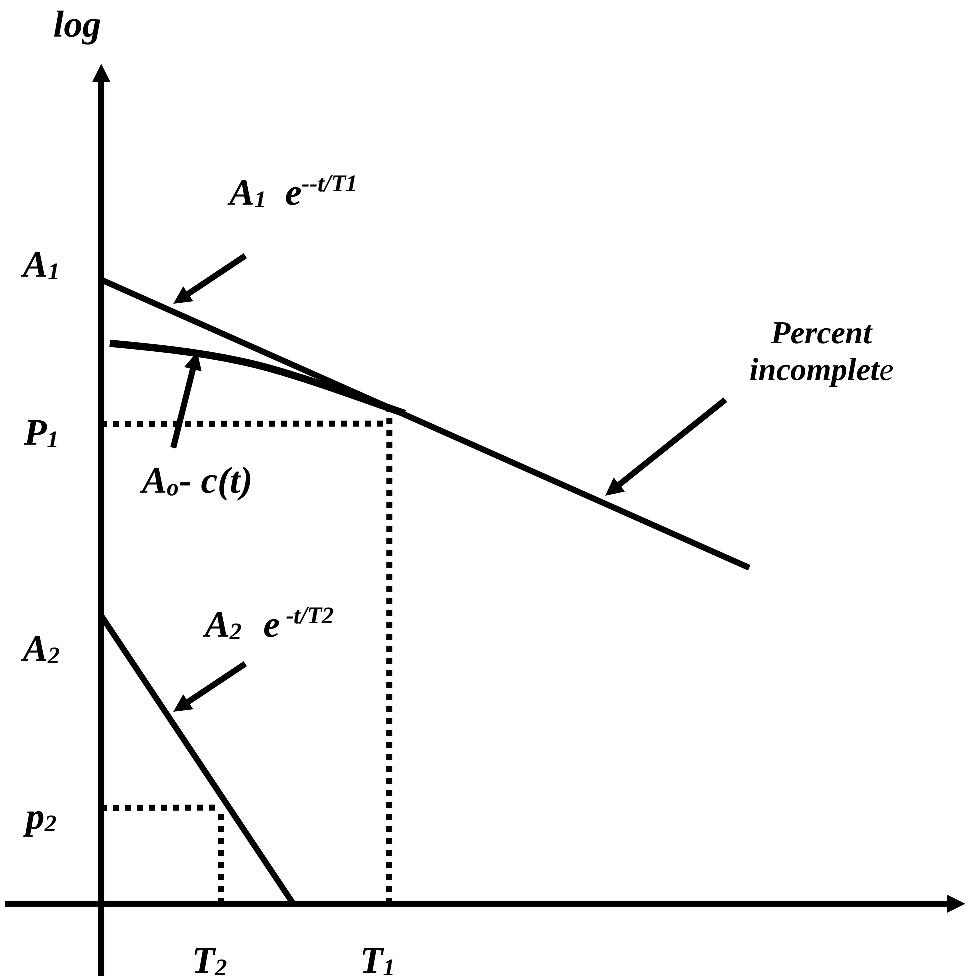
***)** Rearranging the original equation yields

$$A_2 * e^{-t/T_2} = A_1 * e^{-t/T_1} - (A_o - C(t))$$

where

$p_1 = 0.368 *A_1$

$p_2 = 0.368 *A_2$



<u>Example</u>: A unit step response is given by the following table. Use the method of slope intercept to find the T.F of the system.

t	C (t)	$A_o - C(t) \cong A_1 * e^{t/T_1}$	$100* \; A_1 * e^{t/T_1}$
0	0.0	1.0	100
2	0.32	0.68	68
4	0.55	0.45	45
6	0.7	0.3	30
8	0.78	0.22	22
10	0.86	0.14	14
12	0.91	0.09	9
14	0.94	0.06	6
16	0.96	0.04	4
18	0.98	0.02	2
20	1.0	0.0	0

Solution:

$$C(t) = A_o - A_1 * e^{-t/T_1} + A_2 * e^{-t/T_2}$$

At ***t = ∞***

$$C_{S.S} = C(\infty) = A_o - A_1 * e^{-\infty} + A_2 * e^{-\infty} = A_o$$

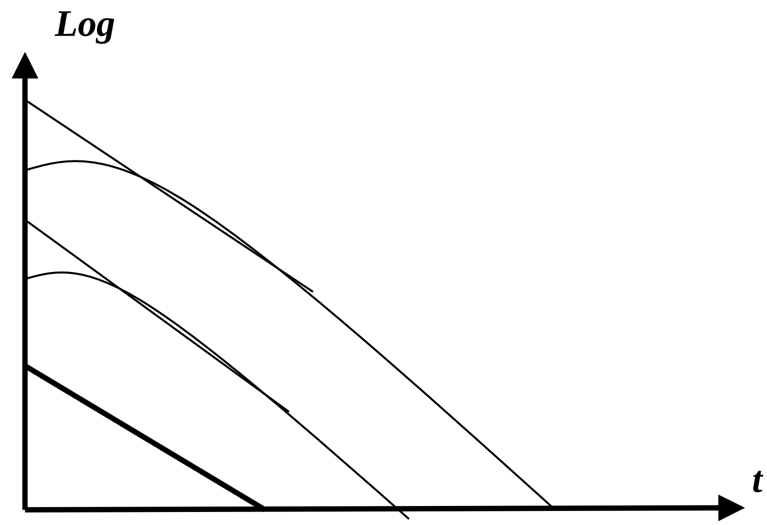
The output at steady state = $\boldsymbol{A_o}$ =1, so the ***Gain= $C_{S.S}$ ÷ I/P = 1***

For $T_1 > T_2$ and $t >> T_2$

$$A_o - C(t) \cong A_1 * e^{-t/T_1}$$

$$G(s) = \frac{k}{(s + \frac{1}{T_1})(s + \frac{1}{T_2})} = \frac{1}{(s + \frac{1}{4.8})(s + \frac{1}{2.4})}$$

Note: If the (extrapolation – percent incomplete line) is a straight line then the system is a 2nd order or if not (i.e. curved) extrapolate again and the system is a higher order



Home work:

Q1/ A unit step response of a 2nd order system is shown below. Estimate the time constants of the system using slope intercept method.

t	0	2	4	6	8	10	12	14	16
C(t)	0	.375	.625	.75	.85	.925	.95	.975	.98

Q2/ Determine the 2nd order system parameters from the following open loop system response data.

t	0	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.7	1	1.2
C(t)	0	.56	.98	1.3	1.54	1.72	1.86	2.04	2.14	2.23	2.27	2.27

Q3/ Estimate the transfer function of a system from the open loop system response data.

t	0	1	2	3	4	5	6	7	8
C(t)	0	.4	.62	.75	.85	.92	.95	.97	.975

A-3 Periodic second order

Periodic second order system can be in general described by:

$$G(s)=\frac{k/w_n^2}{s^2+2\zeta w_n s+w_n{}^2}$$

$$G(s)=\frac{k}{\frac{s^2}{w_n^2}+\frac{2\zeta}{w_n}s+1}$$

***)** where k is the steady state gain = $O/P_{S.S} \div I/P$

***)** $0<\zeta<1$ (under damped system)

***)** $T = 1/w_n$

***)** θ is the period of the first oscillation

***)** $w_d = 2\pi/\theta$ and $w_n=\frac{w_d}{\sqrt{1-\zeta^2}}$

***)** Maximum percent overshoot = $(c(t_p) - c(\infty)) / c(\infty)$ and

***)** $\mu_p = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$