

# Lecture 8

1

$\phi_k$

$$y(k) = -a y(k-1) + b u(k-1) + e(k)$$

Step 1:  $k=1$   $\phi^T(1) = [-y(0) \quad u(0)] = [0 \quad 0.1]$

$$e(1) = y(1) - \hat{\theta}^T(0) \phi(1) = -1.99$$

$$P(1) = P(0) \left[ I - \frac{\text{Num}(1)}{\text{Den}(1)} \right]$$

$$\text{Num}(1) = \phi(1) \phi^T(1) P(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$\text{Den}(1) = \lambda + \phi^T(1) P(0) \phi(1) = 1.1$$

$$P(1) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \left[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \frac{0.1}{1.1} \end{bmatrix} \right] = \begin{bmatrix} 10 & 0 \\ 0 & 9.09 \end{bmatrix}$$

$$\hat{\theta}(1) = \hat{\theta}(0) + P(1) \phi(1) e(1) = \begin{bmatrix} -0.1 \\ 18.28 \end{bmatrix}$$

Step 2:  $k=2$

$$\phi^T(2) = [-0.11 \quad -0.3] \quad e(2) = 5.22$$

$$\text{Num}^{(2)} = \begin{bmatrix} 0.12 & 0.3 \\ 0.33 & 0.82 \end{bmatrix}$$

$$\text{Den}^{(2)} = 1.94$$

$$P(2) = \begin{bmatrix} 9.38 & -1.55 \\ -1.55 & 5.26 \end{bmatrix}$$

$$\hat{\theta}(2) = \begin{bmatrix} -3.08 \\ 10.94 \end{bmatrix}$$

Q2

Step 1:  $k=1$ 

$$\phi^T(1) = [0 \quad -2.1 \quad 1]$$

$$e(1) = y(1) - \phi^T(1) * \hat{\theta}(0)$$

$$= 1.6$$

$$Num = \phi(1) \phi^T(1) P(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 220.5 & -105 \\ 0 & -105 & 50 \end{bmatrix}$$

$$Den = \lambda + \phi^T(1) P(0) \phi(1) = 271.5$$

$$P(1) = P(0) \left[ I - \frac{Num}{Den} \right] = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 9.3923 & 19.337 \\ 0 & 19.337 & 40.7919 \end{bmatrix}$$

$$\hat{\theta}(1) = \hat{\theta}(0) + P(1) \phi(1) e(1)$$

$$= [2.1 \quad -1.6 \quad 1]^T$$

$$e(2) = y(2) - \phi^T(2) \hat{\theta}(1) = 2.3153$$

$$Num = \begin{bmatrix} 220.5 & 9.0497 & 20.6906 \\ -168 & -6.8950 & -15.7643 \\ 105 & 4.3094 & 9.8527 \end{bmatrix}$$

$$Den = 224.4576$$

$$P_{(1)z} = \begin{bmatrix} 0.8816 & -2.0159 & -4.609 \\ -2.0159 & 9.3095 & 19.1479 \\ -4.6090 & 19.1479 & 40.3594 \end{bmatrix}$$

(3)

$$\hat{e}(2) = [1.0831 \quad -0.5743 \quad 0.3963]^T$$

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