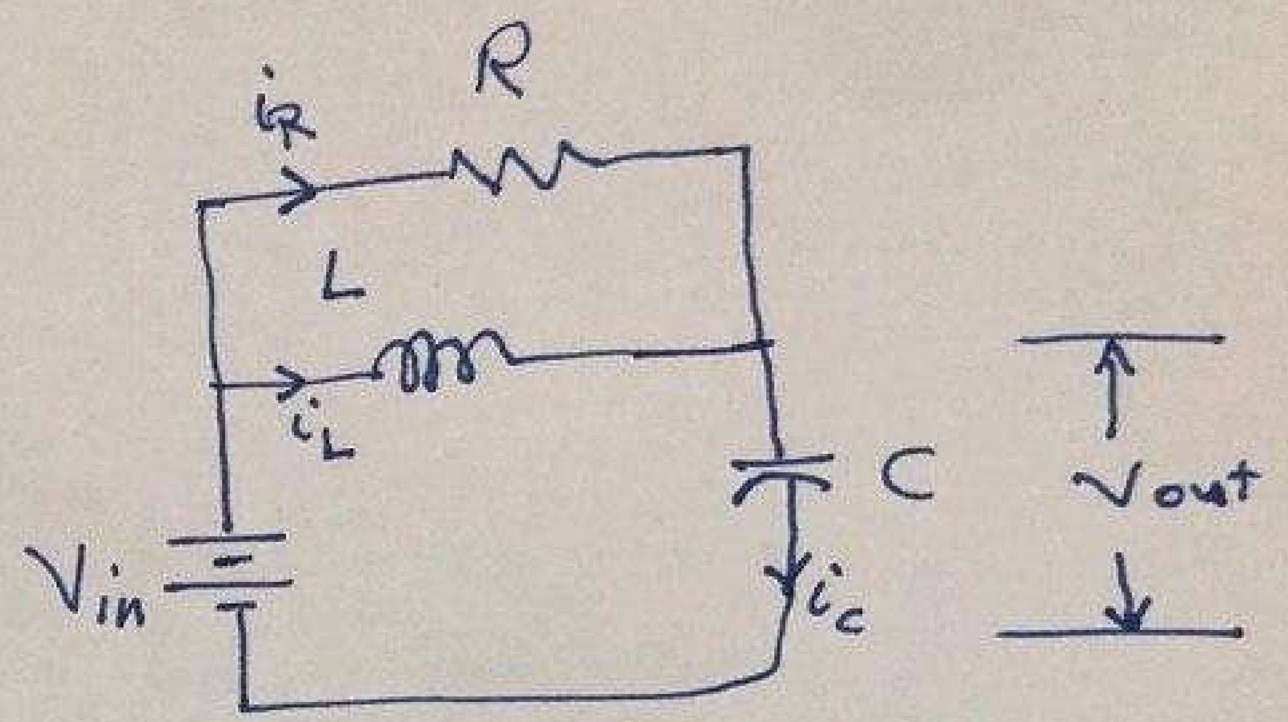
Lec. 1

H.W. 1



$$i_c = c\,\frac{dV_c}{dt}$$

$$V_L = L\,\frac{di_L}{dt}$$

$$V_{out} = V_c = X_1$$

$$V_{out} = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$V_{in} = V_L + V_c$$

$$V_{in} = L\,\frac{di_L}{dt} + V_c$$

let

$$i_L = X_2 \qquad V_c = X_1$$

$$\dot{X}_2 = \frac{1}{L} V_{in} - \frac{1}{L} X_1 \quad \text{—} 1$$

$$V_{in} = V_R + V_c = (i_c - i_L)R + V_c$$

$$V_{in} = i_c R - i_L R + V_c$$

$$= CR\,\frac{dV_c}{dt} - i_L R + V_c$$

$$= CR\dot{X}_1 - RX_2 + X_1$$

$$CR\dot{X}_1 = V_{in} + RX_2 - X_1$$

$$\dot{X}_1 = \frac{V_{in}}{CR} + \frac{X_2}{C} - \frac{X_1}{CR} \quad 2$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{CR} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{CR} \\ \frac{1}{L} \end{bmatrix} V_{in}$$

$$V_{out} = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{-1}{CR} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{CR} \\ \frac{1}{L} \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

$$G(s) = C^T (sI - A)^{-1} B$$

① $$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} \frac{-1}{CR} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} = \begin{bmatrix} s + \frac{1}{CR} & \frac{-1}{C} \\ \frac{1}{L} & s \end{bmatrix}$$

$$\text{cof}\{sI - A\} = \begin{bmatrix} s & \frac{-1}{L} \\ \frac{+1}{C} & s + \frac{1}{CR} \end{bmatrix}$$

$$\text{adj}\{sI - A\} = \begin{bmatrix} s & \frac{1}{C} \\ \frac{-1}{L} & s + \frac{1}{CR} \end{bmatrix}$$

$$\det\{sI - A\} = \left(s + \frac{1}{CR}\right)s + \frac{1}{C} \cdot \frac{1}{L} = s^2 + \frac{s}{CR} + \frac{1}{CL}$$

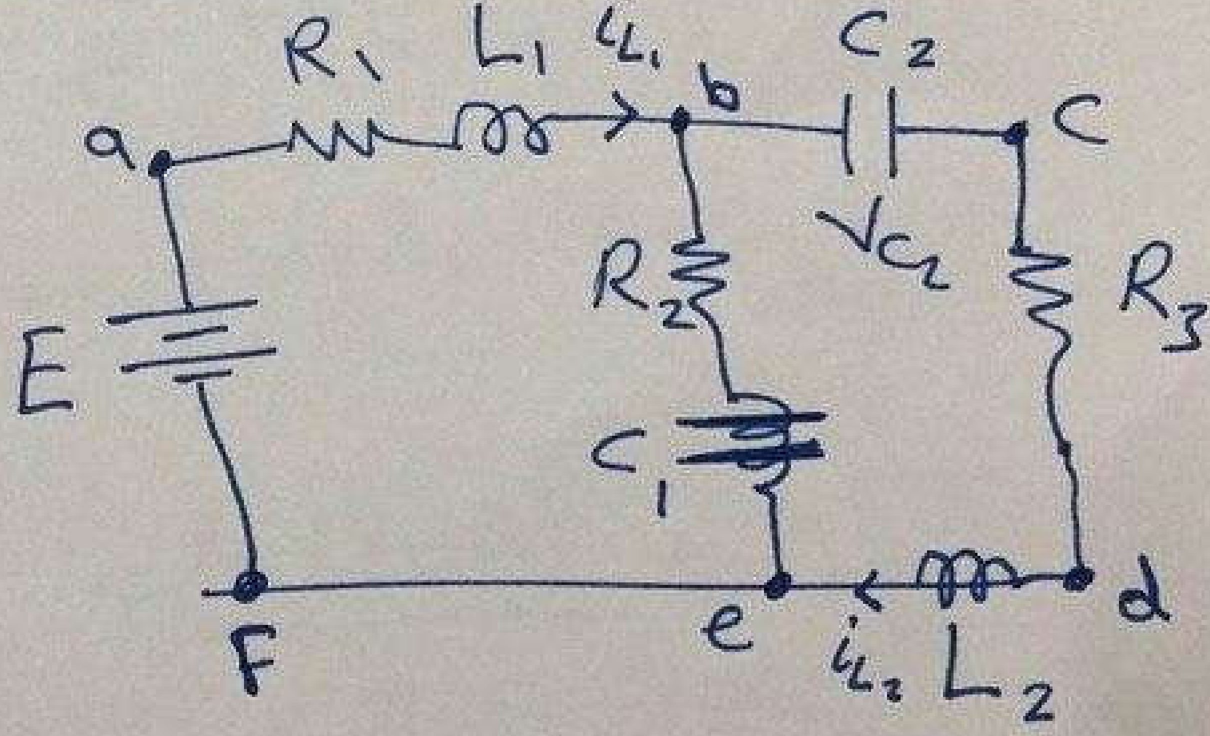
$$(sI - A)^{-1} = \frac{\begin{bmatrix} s & \frac{1}{C} \\ \frac{-1}{L} & s + \frac{1}{CR} \end{bmatrix}}{s^2 + s\frac{1}{CR} + \frac{1}{CL}}$$

$$G(s) = C^T (sI - A)^{-1} B$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \boxed{} & \boxed{} \\ & \end{bmatrix} \begin{bmatrix} \frac{1}{CR} \\ \frac{1}{L} \end{bmatrix}$$

$$G(s) = \frac{\frac{s}{CR} + \frac{1}{CL}}{s^2 + \frac{s}{CR} + \frac{1}{CL}}$$

H.W 2



$V_{C_1} = X_1$

$V_{C_2} = X_2$

$i_{L_1} = X_3$

$i_{L_2} = X_4$

① current in branch be

$$i_{L_1} - i_{L_2} = i_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

$$X_3 - X_4 = C_1 \dot{X}_1$$

$$\dot{X}_1 = \frac{X_3}{C_1} - \frac{X_4}{C_1} \quad \text{---} ①$$

2 current in the second capacitor.

$$i_{L_2} = i_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

$$X_4 = C_2 \dot{X}_2 \Rightarrow \dot{X}_2 = \frac{X_4}{C_2} \quad \text{--- } 2$$

3 loop abefa

$$E = U_{R_1} + U_{L_1} + U_{R_2} + U_{C_1}$$

$$E = i_{L_1} R_1 + L_1 \frac{di_{L_1}}{dt} + (i_{L_1} - i_{L_2}) R_2 + V_{C_1}$$

$$E = X_3 R_1 + \dot{X}_3 L_1 + X_3 R_2 - X_4 R_2 + X_1$$

$$\dot{X}_3 = \frac{E}{L_1} - \frac{X_1}{L_1} - \frac{X_3 (R_1 + R_2)}{L_1} + \frac{X_4 R_2}{L_1} \quad \text{- } 3$$

4 loop bcdeb

$$U_{C_2} + U_{R_3} + U_{L_2} + U_{C_1} + U_{R_2} = 0$$

$$U_{C_2} + i_{L_2} R_3 + L_2 \frac{di_{L_2}}{dt} + V_{C_1} + (i_{L_2} - i_{L_1}) R_2 = 0$$

$$X_2 + X_4 R_3 + L_2 \dot{X}_4 + X_1 + X_4 R_2 - X_3 R_2 = 0$$

5

$$\dot{x}_4 = \frac{-x_1}{L_2} - \frac{x_2}{L_2} + \frac{R_2}{L_2}x_3 - x_4\frac{(R_2+R_3)}{L_2}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{C_1} & -\frac{1}{C_1} \\ 0 & 0 & 0 & \frac{1}{C_2} \\ -\frac{1}{L_1} & -\frac{(R_1+R_2)}{L_1} & 0 & \frac{R_2}{L_1} \\ \frac{-1}{L_2} & \frac{-1}{L_2} & \frac{R_2}{L_2} & -\frac{(R_2+R_3)}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_1} \\ 0 \end{bmatrix} E$$

Lecture 2 H.W

Q1

$$R_{xy}(-2) = \frac{((-2.4)(-0.25) + (1)(3.6) + (-1.75)(1.6))}{3}$$

$$= 0.466$$

$$R_{xy}(3) = \frac{((0.95)(0.5) + (2)(3))}{2} = 3.2375$$

$$R_{yx}(2) = R_{xy}(-2) = 0.466$$

$$R_{yx}(-1) = \frac{((0.5)(3.6) + (3)(1.6) + (-2.4)(0.95) + (1)(2))}{4} = 1.58$$

Q2

$$R_{xy}(-3) = \frac{((1.6)(1)+(3)(1)+(3.4)(2)+(4)(3)+(2)(2))}{5}$$

$$= 5.48$$

$$R_{yy}(3) = \frac{((3)(1)+(2)(1)+(1)(2)+(3)(3)+(4)(2)]}{5} = 4.8$$

$$R_{yx}(2) = \frac{((0.4)(1)+(1.6)(1)+(3)(2)+(3.4)(3)+(4)(2)+(2)(1))}{6}$$

$$= 4.7$$

$$R_{xy}(3) = \frac{((3)(0.1)+(2)(0.3)+(1)(0.4)+(3)(1.6)+(4)(3))}{5}$$

$$= 3.62$$

Q3

$$R_{xy}(1) = \frac{((0.4)(-2.2)+(-3.2)(-1.8)+(-0.8)(-2.2)+(1.4)(-1.8))}{4}$$

$$= 1.03$$

$$R_{yx}(1) = \frac{((-3.2)(-0.8)+(-2.2)(-2.2)+(1.4)(-1.8)+(3.4)(-0.8))}{4}$$

$$= 0.54$$

$$R_{yy}(2) = \frac{((-1.8)(-0.8)+(-0.8)(-2.2)+(-1.8)(-1.8))}{3} = 2.146$$

$$R_{xx}(-2) = \frac{((-2.2)(0.4)+(1.4)(-3.2)+(3.4)(-2.2))}{3} = -4.28$$

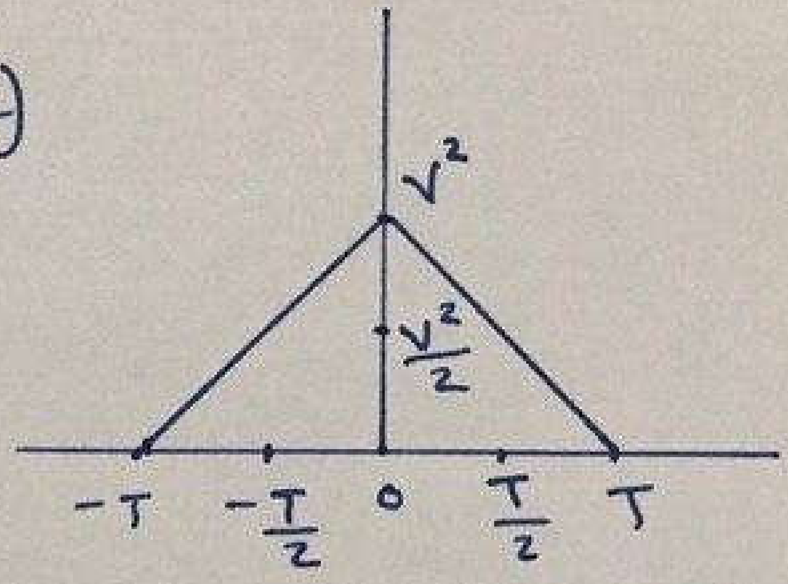
Q4 $R_{xx}(\tau) = V^2\left[1 - \frac{|\tau|}{T}\right]$ $\quad -T \leq \tau \leq T$

① at $\tau = 0$ $\quad R_{xx} = U^2$

② at $\tau = \pm\frac{T}{2}$ $\quad R_{xx} = U^2\left(1 - \frac{T/2}{T}\right)$

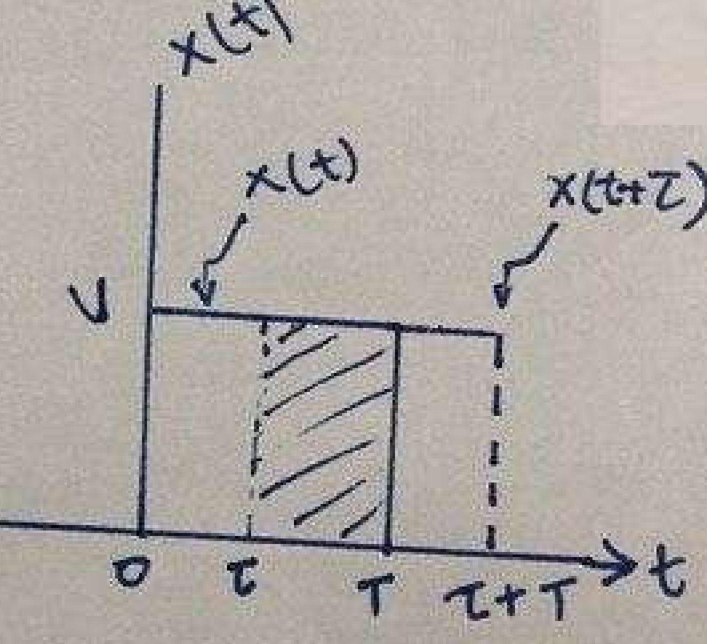
$= \frac{U^2}{2}$

③ at $\tau = \pm T$ $\quad R_{xx} = 0$



Q5 $R_{xx}(\tau) = \frac{1}{T}\int_0^T x(t)\, x(t+\tau)\, dt$

$= \frac{1}{T}\left[\int_0^\tau 0 * U\, dt + \int_\tau^T U^2\, dt\right]$



$R_{xx}(\tau) = \frac{U^2}{T}(T - \tau) \quad = U^2\left(1 - \frac{|\tau|}{T}\right)$ $\quad -T \leq \tau \leq T$

Q6

$x(t) = U\sin(\omega t + \theta)$

$R_{xx}(\tau) = \frac{1}{T}\int_0^T \left(U\sin(\omega t + \theta)\right)\left(U\sin(\omega t + \omega\tau + \theta)\right) dt$

using $\sin(x)\sin(y) = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$

$R_{xx}(\tau) = \frac{U^2}{2T}\int_0^T \cos(\omega\tau) - \cos(2\omega t + \cancel{2\theta} + \omega\tau)\, dt$

$= \frac{U^2}{2T}\cos(\omega\tau)\,(T - 0) = \frac{U^2}{2}\cos(\omega\tau)$